Systemic Network Risks in the Russian Interbank Market

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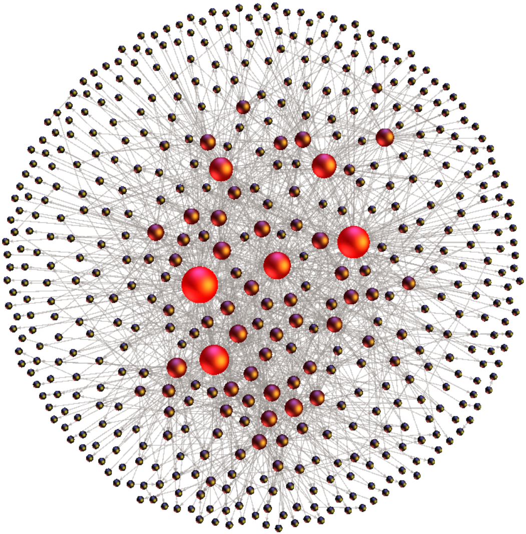
Systemic risks in the interbank market

- Systemic network risks in the interbank market correspond to default (contagion) propagation through mutual obligation network triggered by default of one or several banks (nodes).
- The results presented in this talk are based on joint research with E.L. Rumyantsev:
  - A.L., E.R., "Systemic Interbank Network Risks in Russia", arXiv:1410.0125, Moscow Journal of Combinatorics and Number Theory, in press
  - A.L., E.R., "Default Contagion Risks in Russian Interbank Market", arXiv: 1409.071, Physica A, submitted
  - A.L., E.R., "Estimate of systemic risks of Russian interbank market based on network topology", Journal of NEA 3 (19) (2013), 65-80 (in Russian)
  - A.L., E.R., "Russian interbank networks: main characteristics and stability with respect to contagion", arXiv:1210.3814

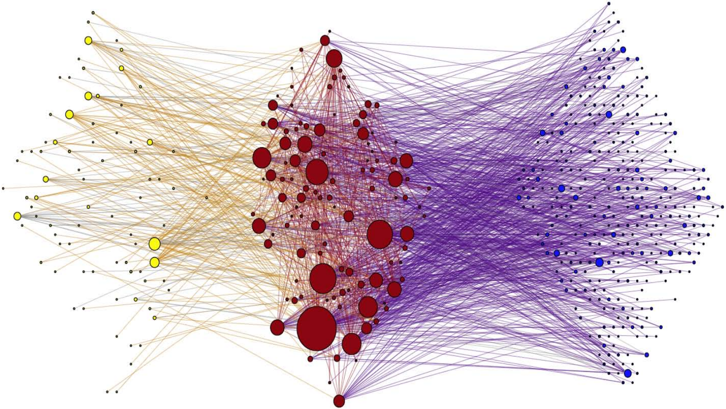
Russian interbank network: data description

- Uncollaterized interbank rouble deposits of all maturities in the period from January 11, 2011 till December 30, 2013 are considered.
- Interbank network for N banks is fully characterized by an oriented weighted graph $G^W = (N, W)$, where $W = \{w_{ij}\}$ is an $N \times N$ matrix of $w_{ij} > 0$ of liabilities of the bank i with respect to the bank j.
- By definition the outgoing links correspond to liabilities, the incoming ones - to claims.
- The interbank network graph is scale-free in both in- and out- degrees and is characterized by significant clustering.

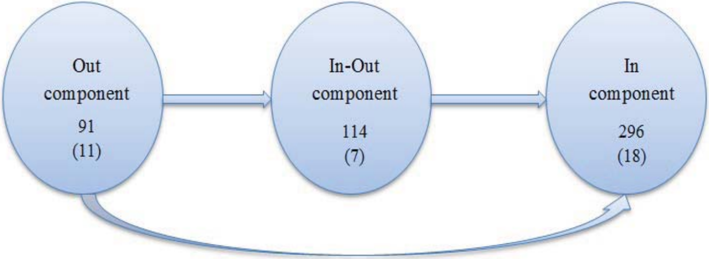
Russian interbank market



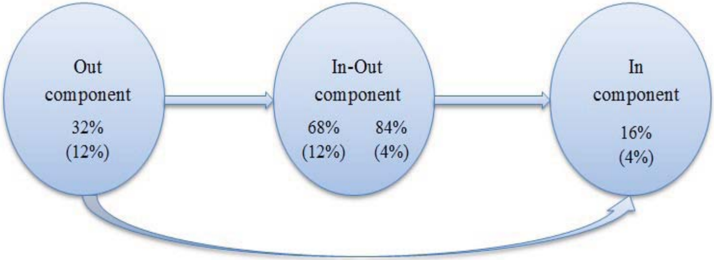
Russian interbank market: bow-tie structure



Interbank network: bow-tie structure: nodes



Interbank network: bow-tie structure: weights



Systemic risks: theoretical modeling

- Mathematical modeling of default propagation uses as its basic input the probability of having at least one incoming link capable of transmitting contagion from adjacent nodes to the node under consideration.
- The probability of default propagation depends on both on the weighted network topology around the node under consideration and characteristics of its balance sheet.
- The choice of mathematical formalism is crucially determined by the characteristic topology of default clusters. Our study shows that, despite of significant clustering of the original network, it is predominantly treelike, so one can use the formalism of generating functions generalized to take explicit account of the bow-tie topology of the interbank network.

Definition of vulnerability

- Solvency coefficient $H1$ as defined by CBRF:

$$H1 = \frac{K}{\sum_i A_i Kp_i + PP + OP + others}$$

- Here
  - K is capital
  - Kp_i - risk coefficients
  - All instruments are divided into 5 groups $i = 1, \cdots 5$ and $Kp_1 = 0$, $Kp_2 = 20$ %, etc. For the interbank market the risk coefficient is 20 %
  - PP - market risk
  - OP - operational risk
  - others - other contributions

Definition of vulnerability

- A default condition is

$$H1 = \frac{K}{\sum_i A_i Kp_i + PP + OP + others} < H1^*$$

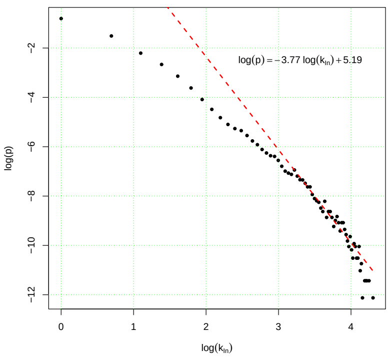
where for banks $H1^* = 10$ %, for others - $H1^* = 12$ %

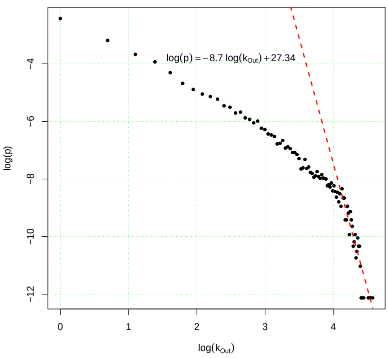
- Calculation using $H1$:

$$H1 \Rightarrow \frac{K - P}{\sum_i A_i Kp_i + PP + OP + others}$$

where P is a reserve kept for the case when one or several counteragents default.

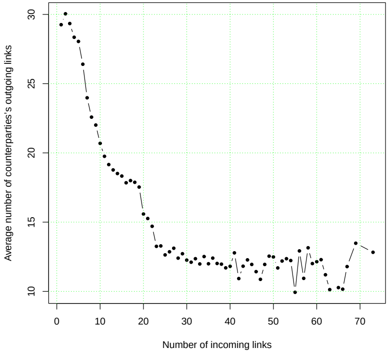
IBN is scale-free: In- and Out- degree distributions

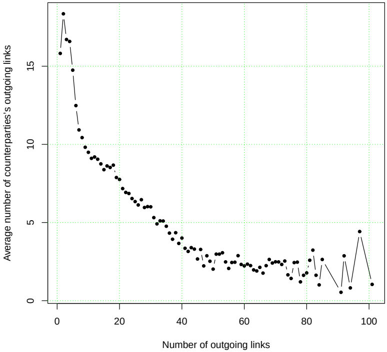




$$P(k_{in}) \sim \frac{1}{k_{in}^{3.77}} \qquad P(k_{out}) \sim \frac{1}{k_{out}^{8.7}}$$

IBN is dissassortative

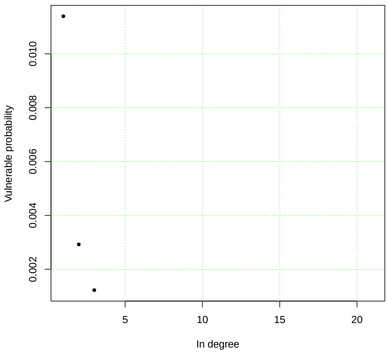




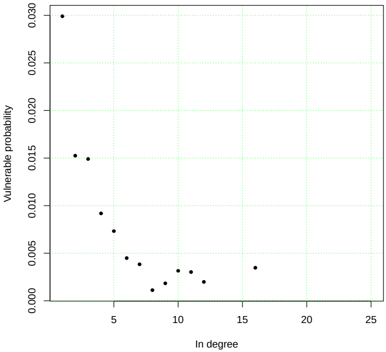
Main input: empirical default distributions

- Probability that at least one incoming link is vulnerable:

Out $\to$ In-Out



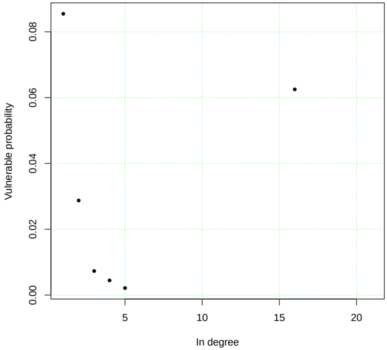
In-Out $\to$ In-Out



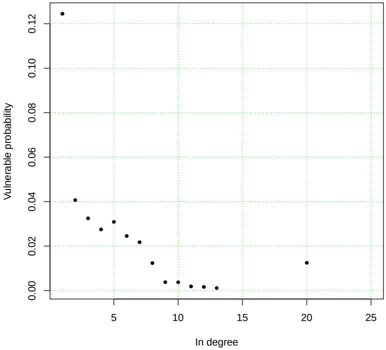
Empirical default distributions

- Probability that at least one incoming link is vulnerable:

Out → In

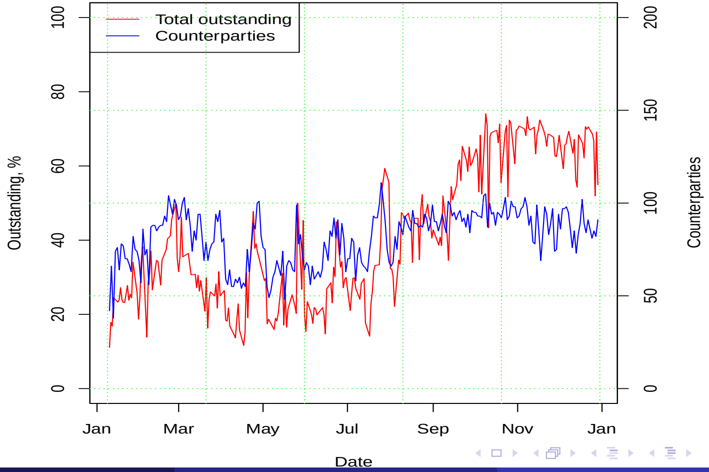


In-Out → In

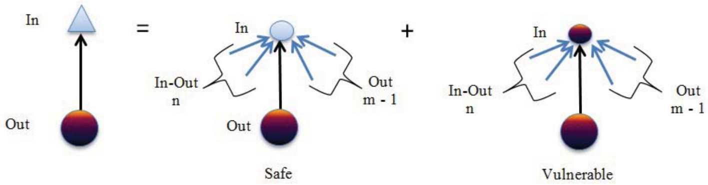


Strongly connected component

- There exists a strongly connected component
- The weight of this component did significantly increase

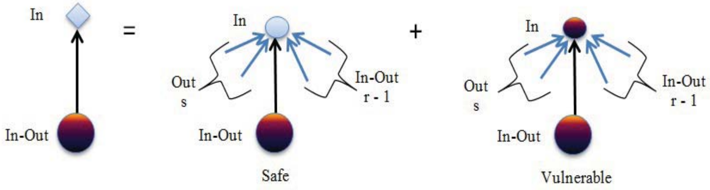


Contagion tree Out → In



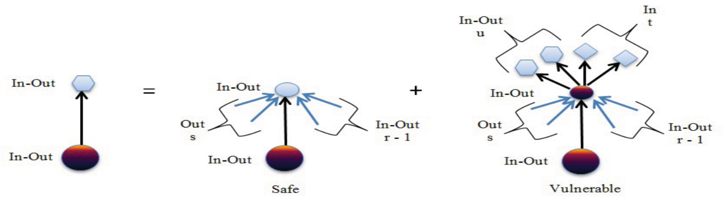
$$L_{i,j}(y) = \sum_{n,m}^{\infty} P_{Out/In}(n, m|i,j) \left[(1 - v_m^{Out/In}) + v_m^{Out/In} y \right]$$

Contagion tree In-Out $\rightarrow$ In



$$N_{k,l}(y) = \sum_{s,r}^{\infty} P_{In-Out/In}(s, r|k, l) \left[(1 - v_r^{In-Out/In}) + v_r^{In-Out/In} y \right]$$

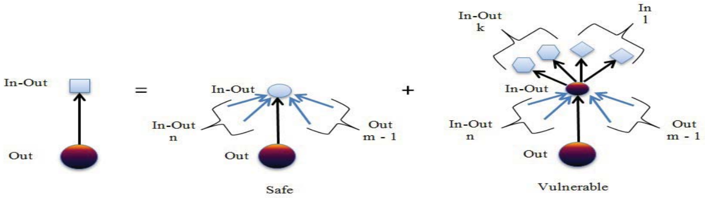
Contagion tree In-Out $\rightarrow$ In-Out & In



$$M_{k,l}(x, N(y)) = \sum_{u,t,s,r}^{\infty} P_{In-Out/In-Out}(u,t,s,r|k,l)$$

$$* \left[(1 - v_r^{In-Out/In-Out}) + x v_r^{In-Out/In-Out} M^u_{u,t}(x, N(y)) N^t_{u,t}(y) \right]$$

Contagion tree Out → In-Out → In-Out & In



$$K_{i,j}(x,y) = \sum_{k,l,n,m}^{\infty} P_{Out/In-Out}(k,l,n,m|i,j)$$

$$* \left[(1 - v_m^{Out/In-Out}) + x v_m^{Out/In-Out} [M_{k,l}(x,y)]^k [N_{k,l}(y)]^l \right]$$

Contagion clusters In-Out → In-Out & In

- Let $F(x, y)$ be the generation function for the probability for a bank from In-Out being linked with In-Out and In components:

$$F(x, y) = \sum_{i,j=0}^{\infty} p_{ij}^{InOut} x^i y^j$$

- The generation function for default cluster originating in In-Out then reads:

$$\mathcal{G}_{InOut}(x, y) = F(M(x, N(y)), N(y))$$

- The average size of default clusters is given by

$$\left. \frac{d\mathcal{G}_{InOut}(x, x)}{dx} \right|_{x=1} = 1$$

Contagion clusters Out → In-Out & In

- Let $G(x, y)$ be the generation function for the probability for a bank from Out being linked with In-Out and In components:

$$G(x, y) = \sum_{i,j=0}^{\infty} p_{ij} x^i y^j$$

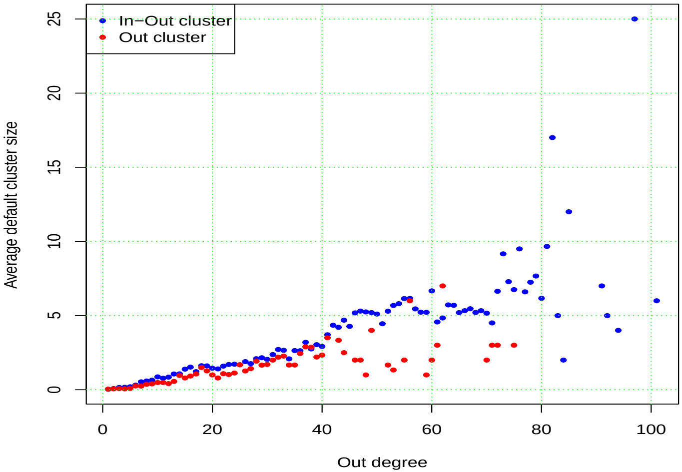
- The generation function for default cluster originating in Out then reads:

$$\mathcal{G}_{Out}(x, y) = G(K(M(x, N(y)), N(y)), L(y))$$

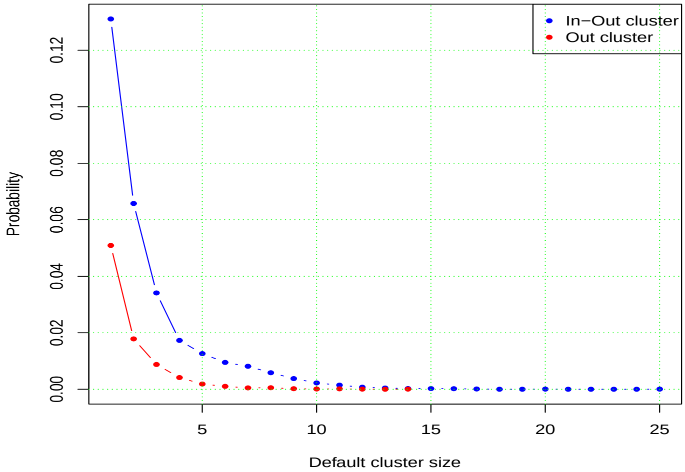
- The average size of default clusters is given by

$$\left. \frac{d\mathcal{G}_{Out}(x, x)}{dx} \right|_{x=1} = 1$$

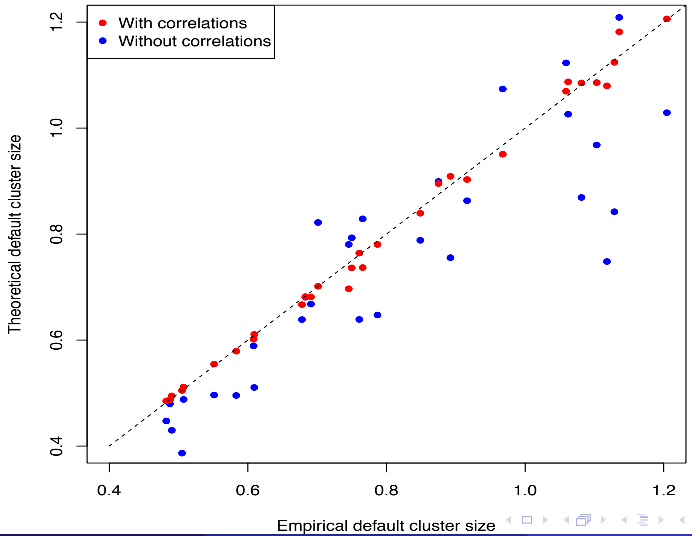
Simulation: dependence upon out-degree



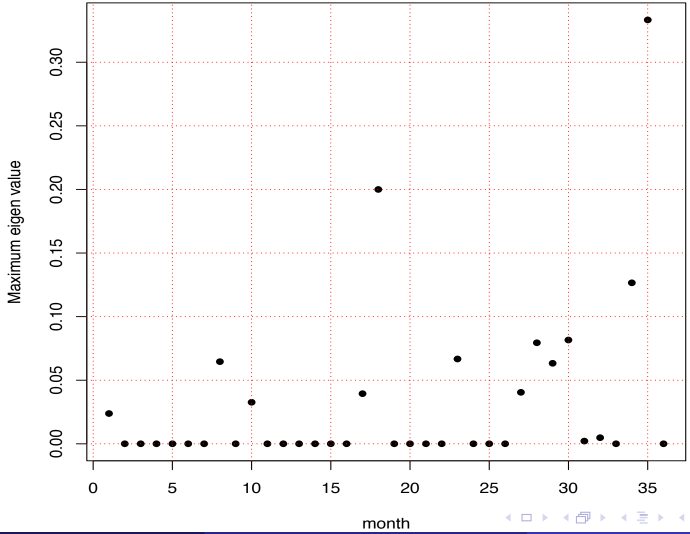
Simulation: default cluster size distribution



Theory: comparison with simulations



Systemic risk: no giant cluster



Conclusions

- Taking into account the bow-tie structure of the interbank network is very essential.
- Despite of the complicated topology of the original graph, the default clusters are (almost) always tree-like.
- This allows to describe default clusters in terms of generating functions taking into account the bow-tie structure of the original interbank network graph.
- The realistic contagion in the RF interbank market is a relatively small effect, nothing dramatic. Giant cluster is not formed.