MARKET MILL DEPENDENCE PATTERN IN HIGH FREQUENCY FINANCIAL SERIES

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Bivariate distribution

- Main goal: quantitative analysis of probabilistic dependence patterns of intraday stock price time series.
- In particular, we will look at two consecutive price increments $x = p(t + \Delta T) - p(t)$ and $y = p(t + 2\Delta T) - p(t + \Delta T)$, fully characterized by bivariate distribution $\mathcal{P}(x, y)$.

Conditional distribution and its moments

- Bivariate distribution is fully characterized by the conditional and marginal ones

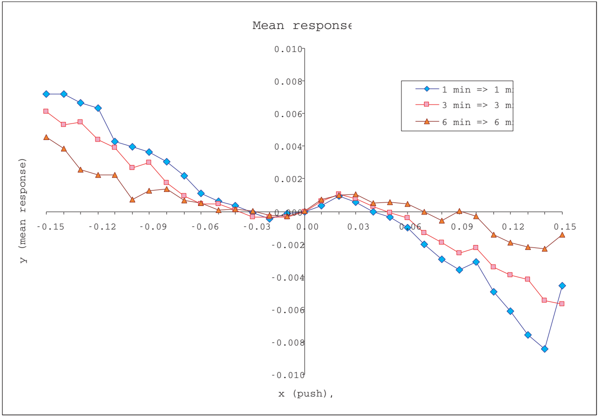
$$\mathcal{P}(x, y) = \mathcal{P}(y|\, x) \cdot \mathcal{P}(y)$$

- Conditional distribution determines conditional dynamics

$$\mathrm{AR}(1) - \mathrm{ARCH}(1) \;\; \Rightarrow \;\; \mu_y \sim r_x, \quad \sigma_y^2 = \alpha + \beta r_x^2$$

Conditional mean

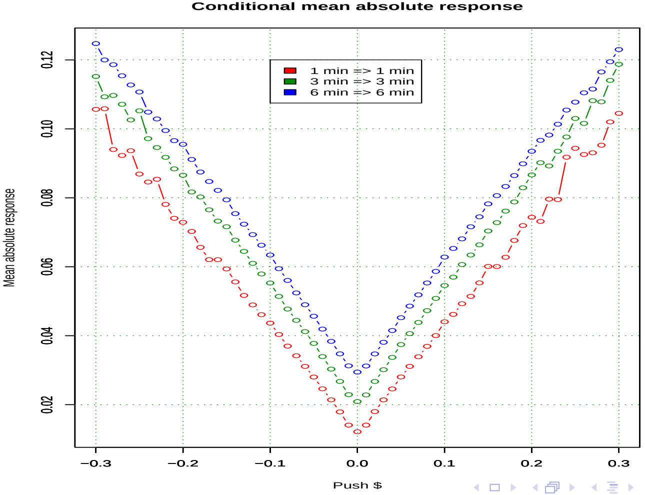
Conditional mean $\langle y \rangle_{x=x_0} = \int dy\; y\; \mathcal{P}(y|\, x = x_0)$



- At small x_0 the response is trendlike.
- At large x_0 the response is contrarian.

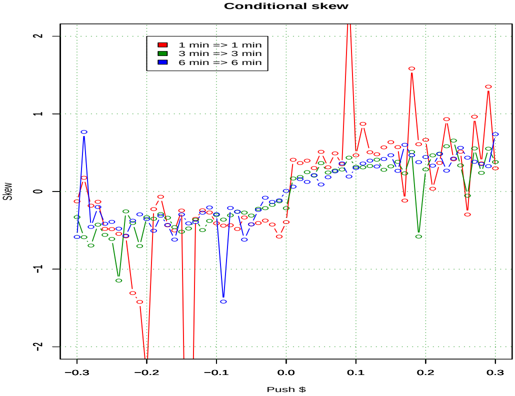
Conditional volatility

Conditional volatility $\langle|\, y|\rangle_{x=x_0} = \int dy\, |\, y|\, \mathcal{P}(y|\, x = x_0)$



Conditional skewness

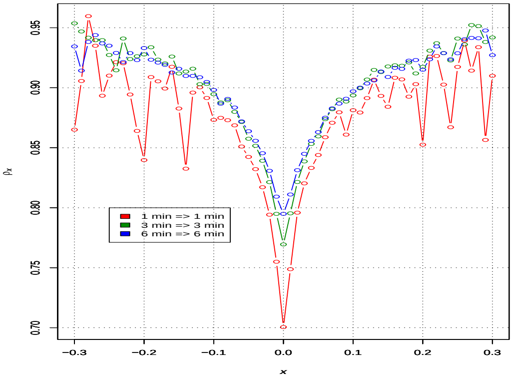
Conditional skewness $\mathcal{V}(x) = \frac{1}{(\sigma_x^y)^3} \int dy \, (y - \langle y \rangle_x)^3 \, P(y|\, x)$



- The sign of conditional skewness inherits the one of the push

Non-gaussianity

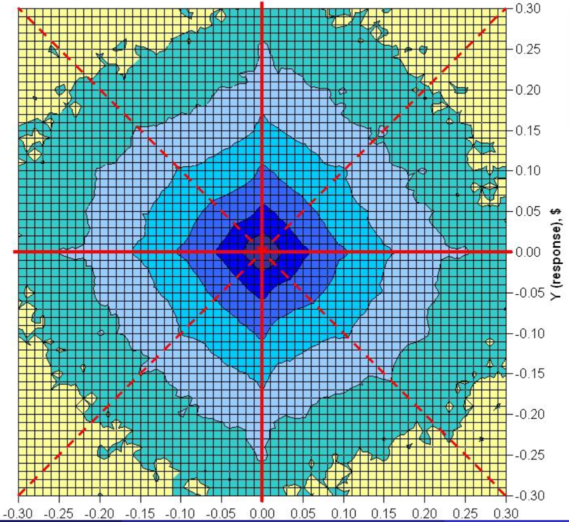
The natural measure of non-gaussianity: $\rho_x = \frac{\langle |y| \rangle_x}{\langle |y| \rangle_x^G}$



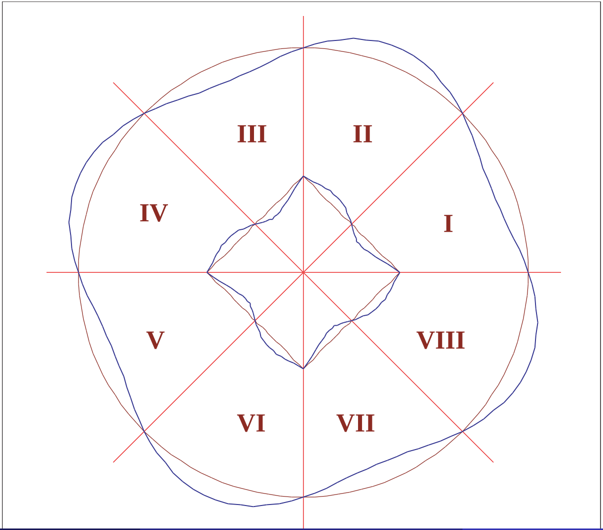
- Conditional distribution is more gaussian with growing x!

Bivariate distribution

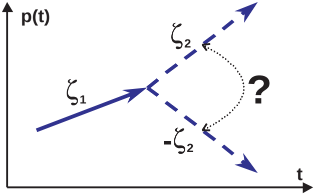
Bivariate distribution $\mathcal{P}(y, x)$



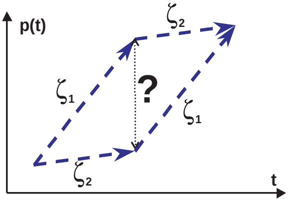
Bivariate distribution $\mathcal{P}(y, x)$: a sketch



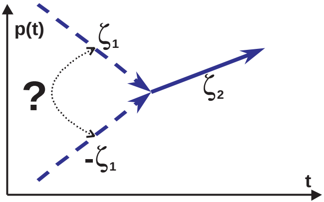
Asymmetric conditional response and non-commutativity



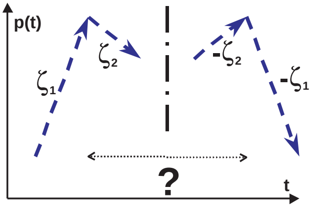
$$\mathcal{P}(\xi_1, \xi_2) \neq \mathcal{P}(\xi_1, -\xi_2))$$



Asymmetric conditional push and path dependence



$$\mathcal{P}(\xi_1, \xi_2) \neq \mathcal{P}(-\xi_1, \xi_2))$$



Market mill asymmetry of $\mathcal{P}(y,x)$

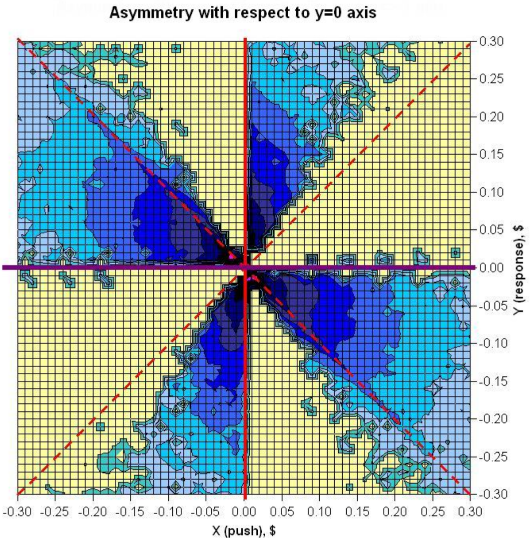
- Symmetric and antisymmetric components with respect to $y = 0$:

$$\begin{aligned} \mathcal{P}(x,y) &\equiv \mathcal{P}^s(x,y) + \mathcal{P}^a(x,y) \\ \mathcal{P}^s(x,y) &= \frac{1}{2}\left(\mathcal{P}(x,y) + \mathcal{P}(x,-y)\right) \\ \mathcal{P}^a(x,y) &= \frac{1}{2}\left(\mathcal{P}(x,y) - \mathcal{P}(x,-y)\right) \end{aligned} \tag{1}$$

- Mill asymmetry

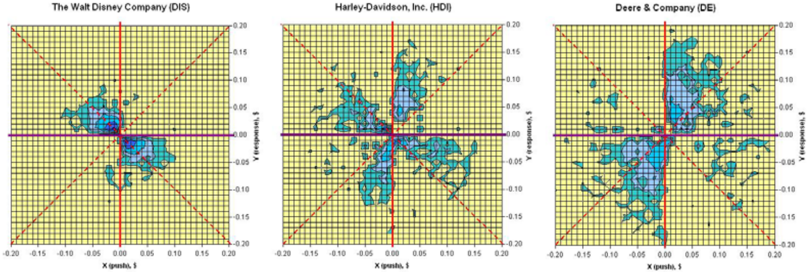
$$\mathcal{P}_{\mathrm{mill}}(x,y) \equiv \mathcal{P}^a(x,y) \cdot \Theta\left[\mathcal{P}^a(x,y)\right] \tag{2}$$

Market mill

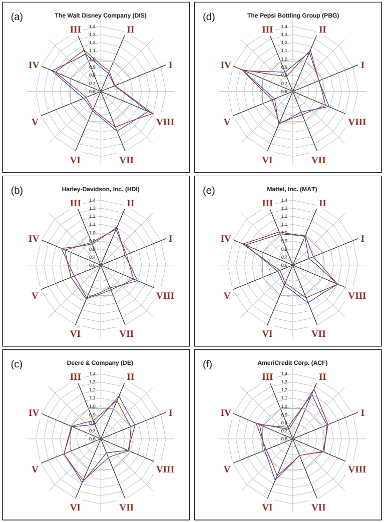


Asymmetry with respect to y=0 axis
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
Y (response), $
-0.30 -0.25 -0.20 -0.15 -0.10 -0.05 0.00 0.05 0.10 0.15 0.20 0.25 0.30
X (push), $

Types of asymmetries for individual stocks:



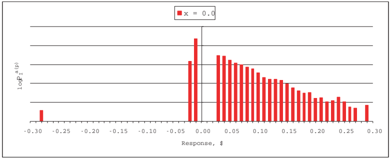
Stability of asymmetry profiles

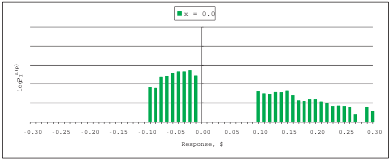


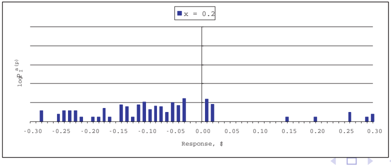
Market mill

- Market mill is responsible for a number of nontrivial properties of $\mathcal{P}(y, x)$. For example, $\mathcal{P}(x_0, y_0) \neq \mathcal{P}(y_0, x_0)$.
- Dynamically market mill emerges through asymmetric component of $\mathcal{P}_{\mathrm{mill}}(y|\, x = x_0)$ for which an explicit multiscale model can be written.
- In particular, market mill is responsible for the z-shaped $\langle y \rangle_{x=x_0}$.

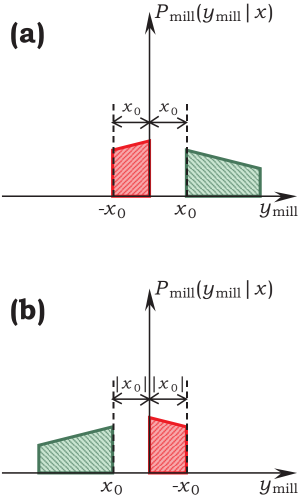
Observed conditional distribution







Conditional dynamics I

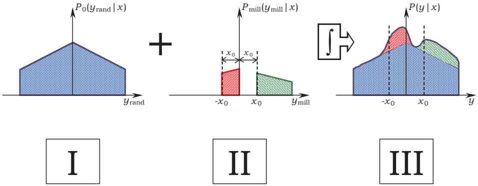


Conditional dynamics II

$$\mathcal{P}(y|\,x) = \int dy_{\rm rand}\, dy_{\rm mill}\, \delta(y - y_{\rm rand} - y_{\rm mill})\, \mathcal{P}_0(y_{\rm rand}|\,x)\, \mathcal{P}_{\rm mill}(y_{\rm mill}|\,x)$$

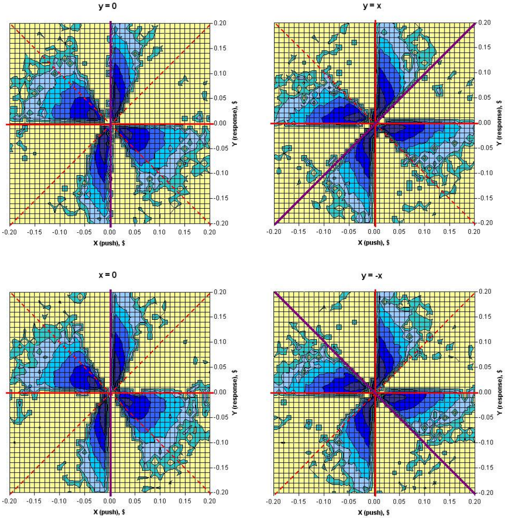
$$y_{\rm rand} \quad + \quad y_{\rm mill} \quad = \quad y$$

$$P_0(y_{\rm rand}\,|\,x) + P_{\rm mill}(y_{\rm mill}\,|\,x) \;\boxed{\int}\!\Rightarrow P(y\,|\,x)$$

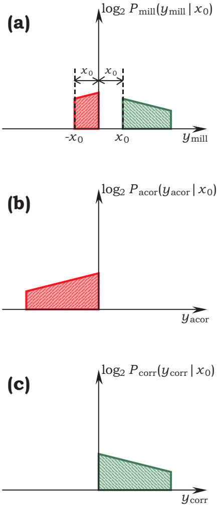


Conditional dynamics

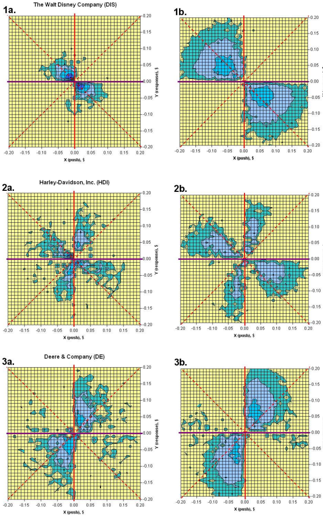
Result:



On the way to individual portraits: mixture of strategies



Comparison to the data:



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