Equity portfolio liquidation value estimation with microstructure taken into account (MICEX case)

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Traditional Approach

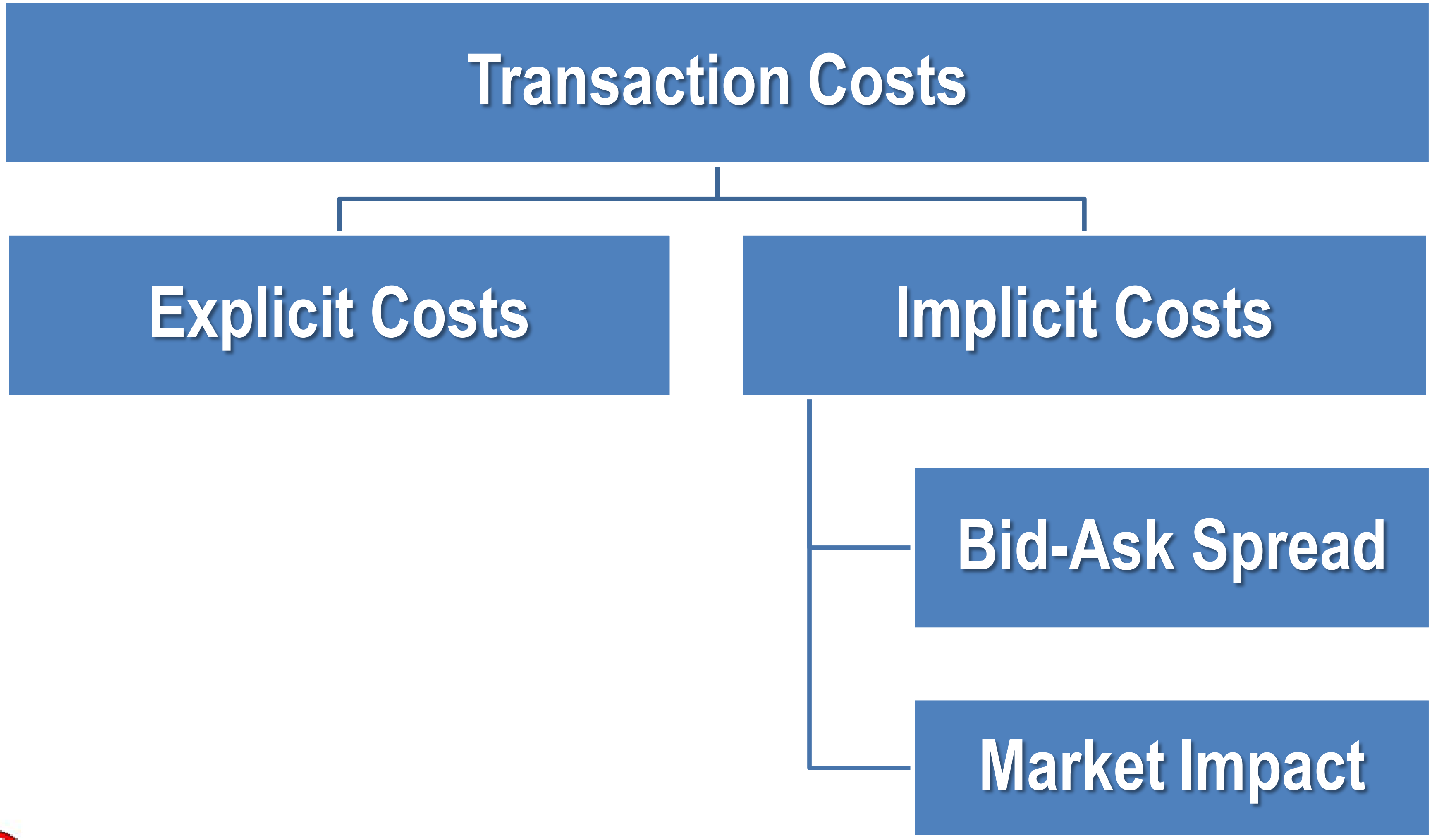
Perfect market:

- ➢Security markets are perfectly elastic (traders act as price takers).
- ➢All market orders have immediate execution.

Efficient market:

- ➢Prices reflect all available information (***Fama, 1970***).
- ➢Price adjustment process.

Transaction Costs



Portfolio Management

- Optimal impulse control.
- Market liquidity.
- Liquidation value.

Market Microstructure Models

- Traditional models (CAPM, Black-Scholes etc.) do not account market in asset pricing ("Black box").

- Market Microstructure is the study of the process and outcomes of exchanging assets under explicit trading rules (*O'Hara, 1995*).

Market Liquidity

- Slippery and elusive concept, reflective its multi-faceted nature.
- A liquid market is a market where participants can *rapidly* execute *large-volume* transactions with a *small* impact on prices (*BIS, 1999*).
- Liquidity is a relative market characteristic.
- Market design defines liquidity.

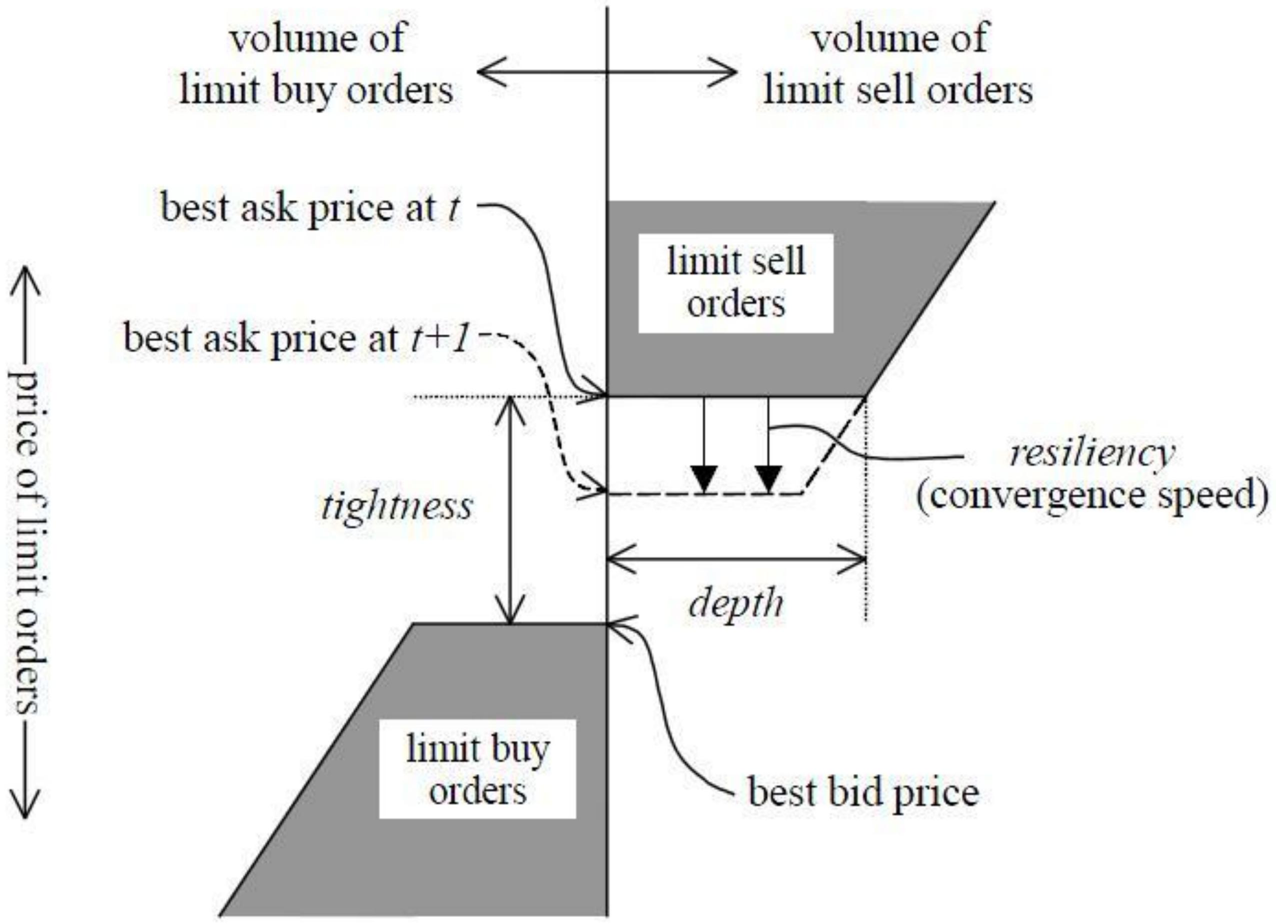
Market Types (Execution Systems)

Order-driven market
Quote-driven market Price-driven market Dealer market
Brokered market

Aspects of Market Liquidity (Kyle, 1985)

- Tightness – the cost of turning around a position over a short period of time.
- Depth – the size of an order flow innovation required to change prices a given amount.
- Resiliency – the speed with which prices recover from a random, uninformative shock.

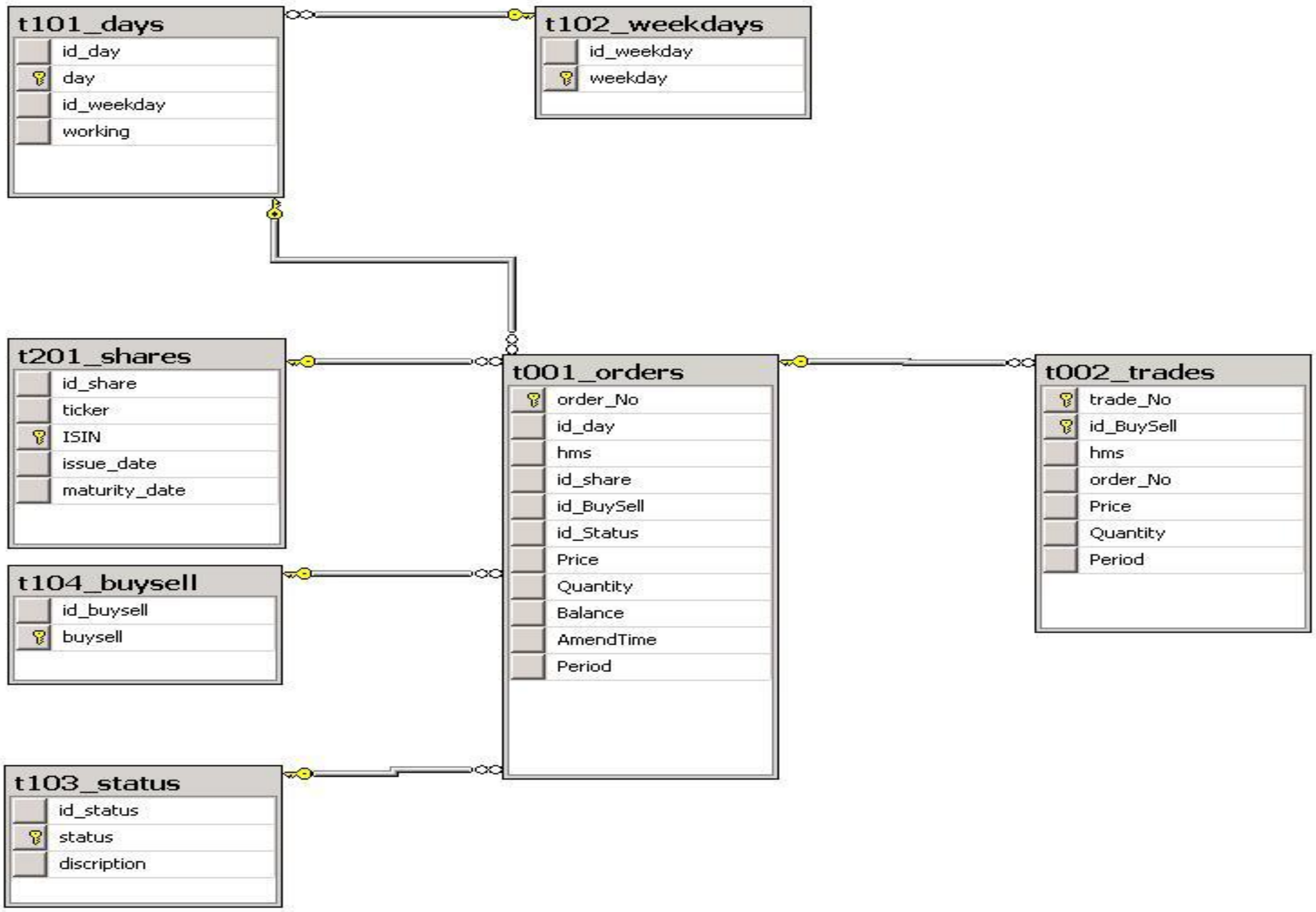
Dimensions of Market Liquidity



Data Set

- **708** stocks traded in MICEX from January 2006 till December 2007.
- All orders during this period (***160 115 507 records***).
- All transactions during this period (***151 022 856 records***).

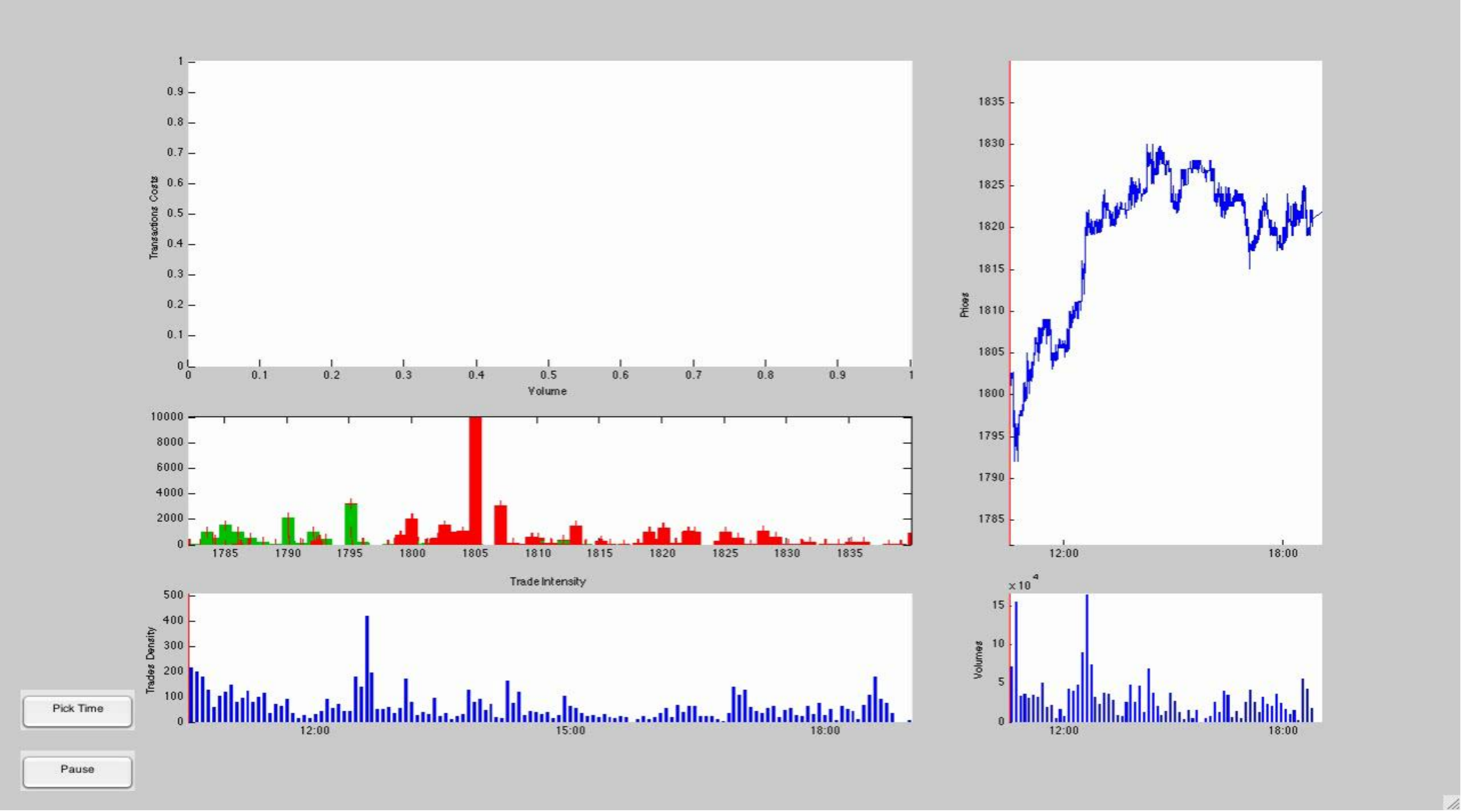
Database Structure



Data Preparation

- Limit order book replication (*Kavajecz, 1999*).
- Data modification.
- Aggregation (median trade volume, bid-ask spread etc.).
- Splitting orders into several types for market resiliency estimation.

Visualization of Trading Process



Cost Function

$$\Theta_t(n_k) = \sum_{i=1}^{N} (p_i - p) n_i$$

n_k – total volume to be executed in time *k*,

p_i – execution price of i th order,

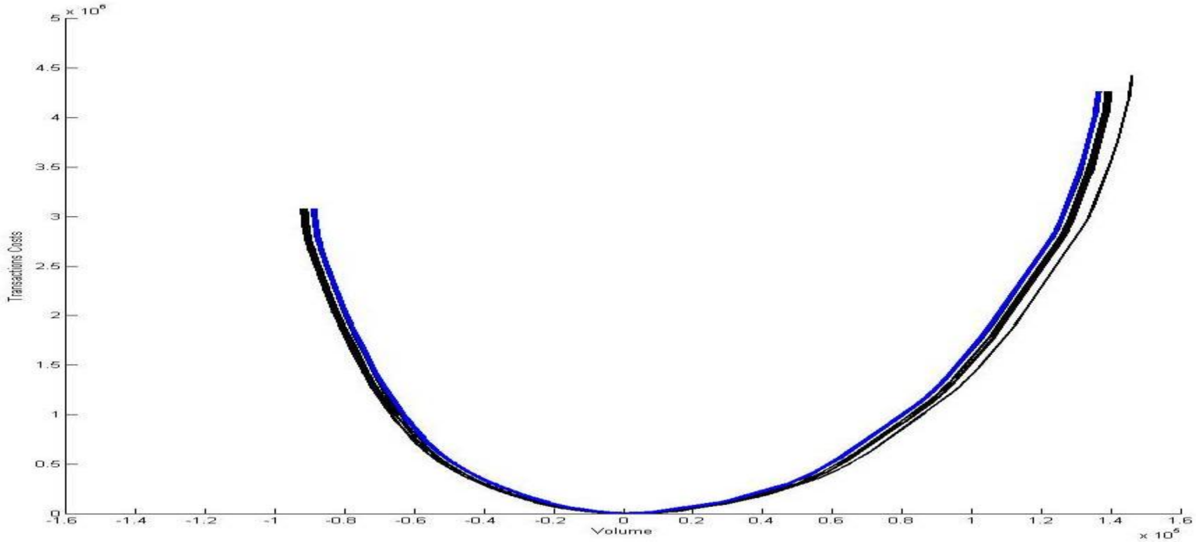
n_i – volume of *i* th order,

p – asset market price which is the following:

$$p = \frac{Bid + Ask}{2}$$

Characteristics of Cost Function

- Accounts for tightness and depth.
- Random non-negative convex function with f(0)=0.
- Approximated by ax^3+bx^2.



Portfolio Liquidation Value

- Portfolio value is a liquidation value.
- Portfolio liquidation depends on liquidation strategy:
  - Immediate liquidation.
  - Uniform liquidation.

- Optimal portfolio liquidation can be defined by different ways including hedging.

Optimal Liquidation Model

V – initial volume,

v_k – volume for liquidation in time k,

V_k – volume left at the beginning of time k,

μ – average trend in time k,

σ – volatility,

$E\Theta(v_k)$ – average costs in time k,

$Var\Theta(v_k)$ – variance of costs in time k

Optimal Liquidation Task

$$
\begin{cases}
\sum_{i=1}^{N} v_i = V \\
EW = x_0 V + \mu \sum_{k=1}^{N} \sum_{i=k}^{N} v_i - \sum_{k=1}^{N} E\Theta_k(v_k) \\
VarW = \sigma^2 \sum_{k=1}^{N} \left(\sum_{i=k}^{N} v_i \right)^2 + \sum_{k=1}^{N} Var\Theta_k(v_k) \\
EW \to \sup\limits_{v_1,\dots,v_N} \\
VarW \to \inf\limits_{v_1,\dots,v_N}
\end{cases}
$$

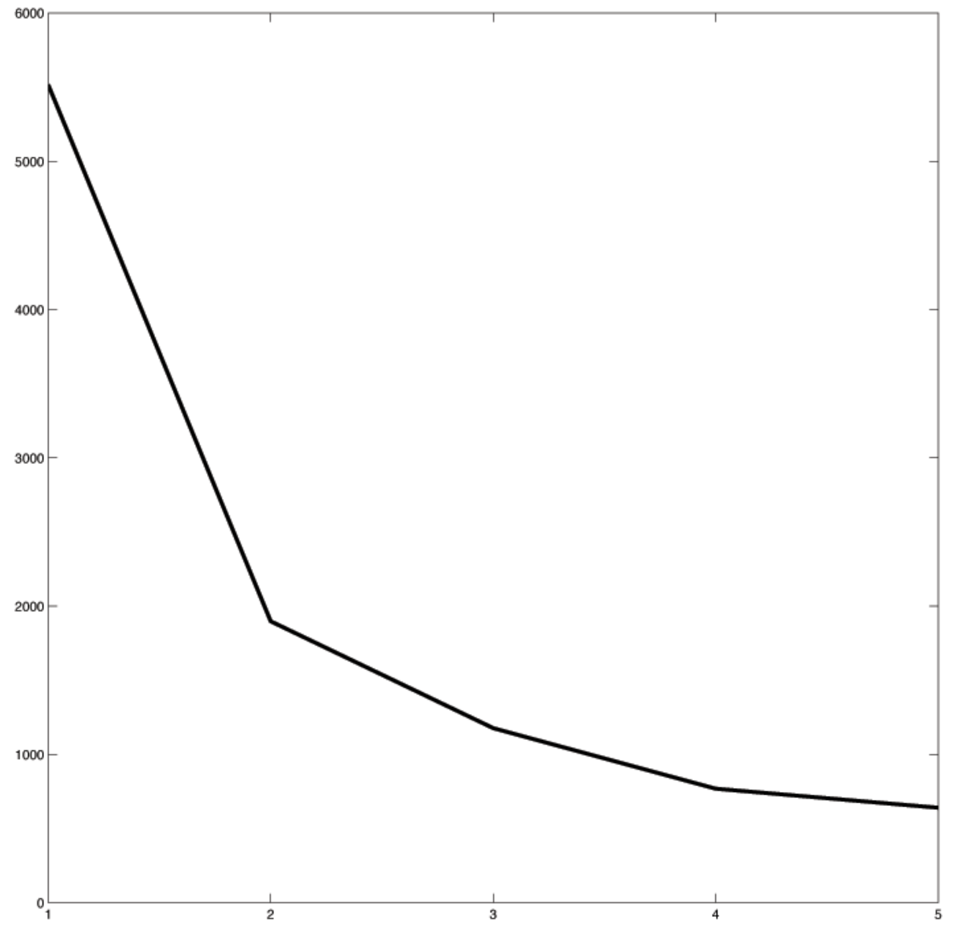
Optimal Liquidation Strategy

- $$J(v)=\lambda\sqrt{VarW}-E(W)\to\inf_{v_1,\dots,v_n}$$
- λ – risk aversion coefficient;
- $\lambda=0$ – risk-neutral (uniform liquidation);
- $\lambda>0$ – risk-averse;
- $\lambda=\infty$ - immediate liquidation.

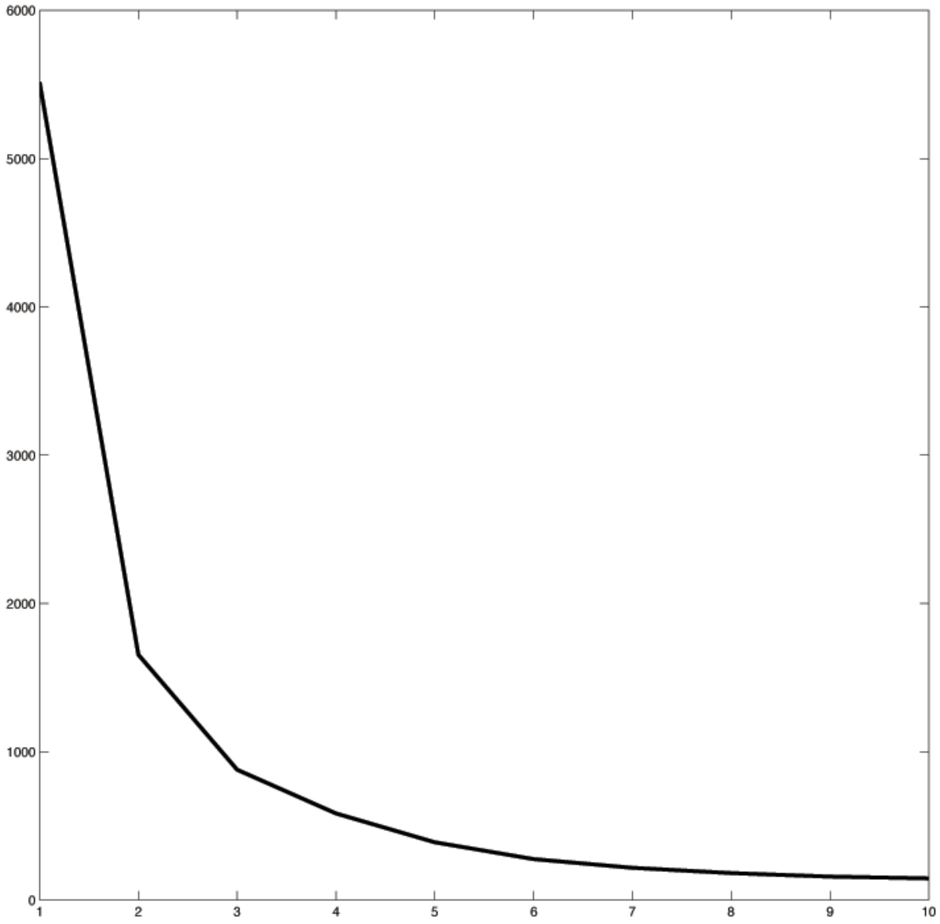
Time Horizon Dependency

- The larger the number of time intervals the less aggressive the trading.
- The largest part of total volume is liquidated in first time intervals.

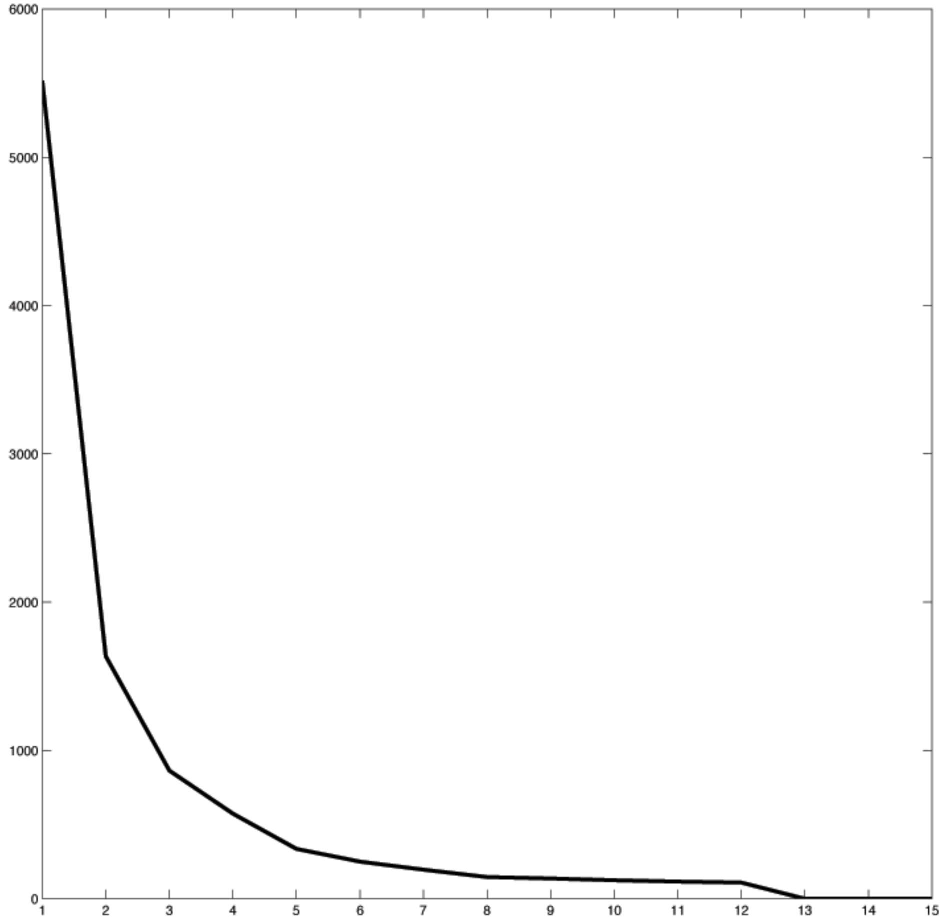
Portfolio Liquidation (Lukoil, 10000 stocks, N=5, λ=1)



Portfolio Liquidation (Lukoil, 10000 stocks, N=10, λ=1)



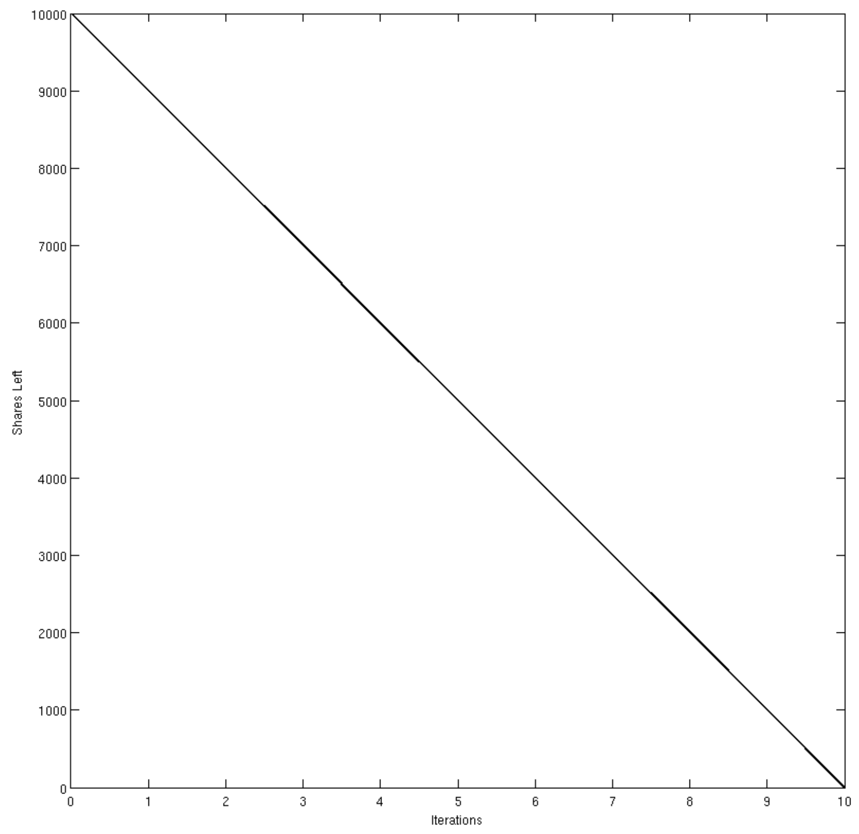
Portfolio Liquidation (Lukoil, 10000 stocks, N=15, λ=1)



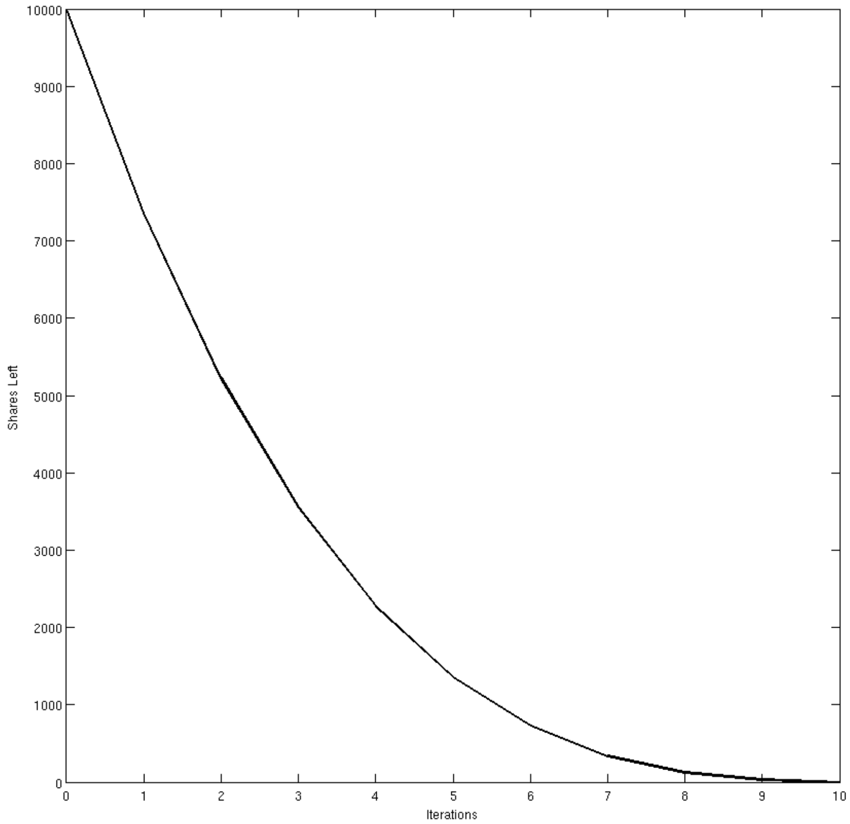
Risk Aversion Coefficient Dependency

- The larger the risk aversion coefficient the more aggressive the liquidation.
- $\lambda=0$ – risk-neutral (uniform liquidation);
- $\lambda>0$ – risk-averse;
- $\lambda=\infty$ - immediate liquidation.

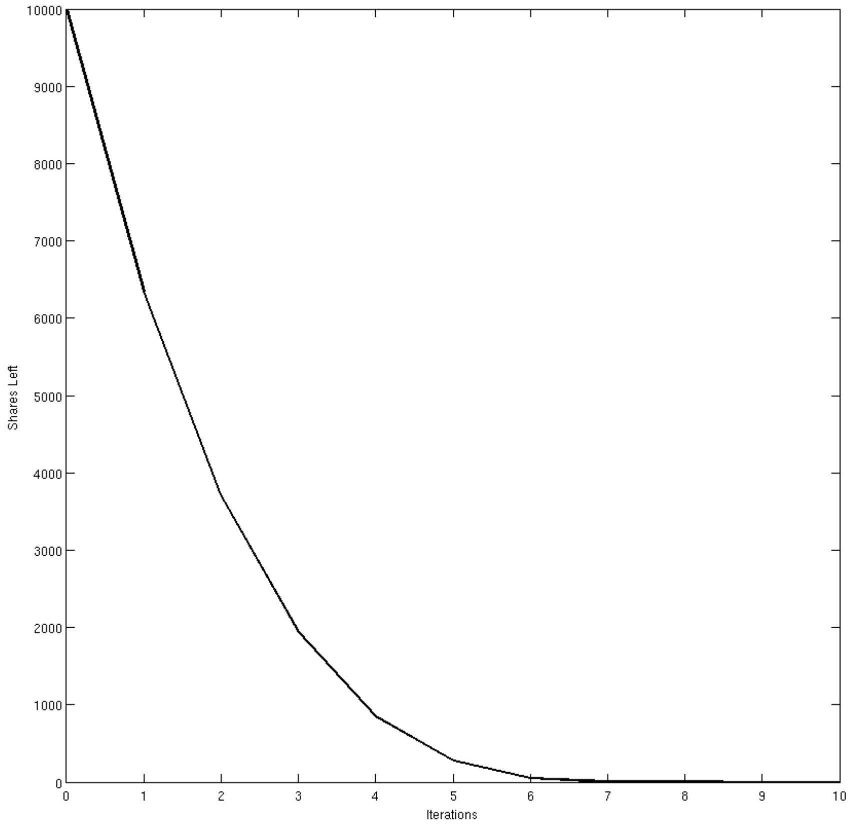
Portfolio Liquidation (Lukoil, 10000 stocks, N=10, λ=0)



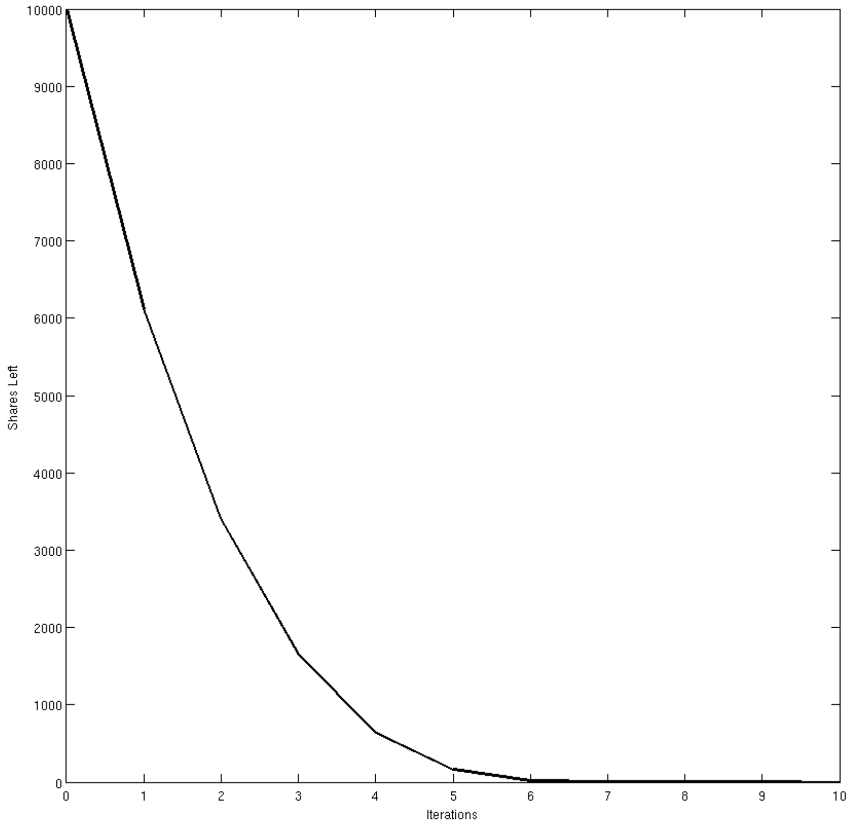
Portfolio Liquidation (Lukoil, 10000 stocks, N=10, λ=0,1)



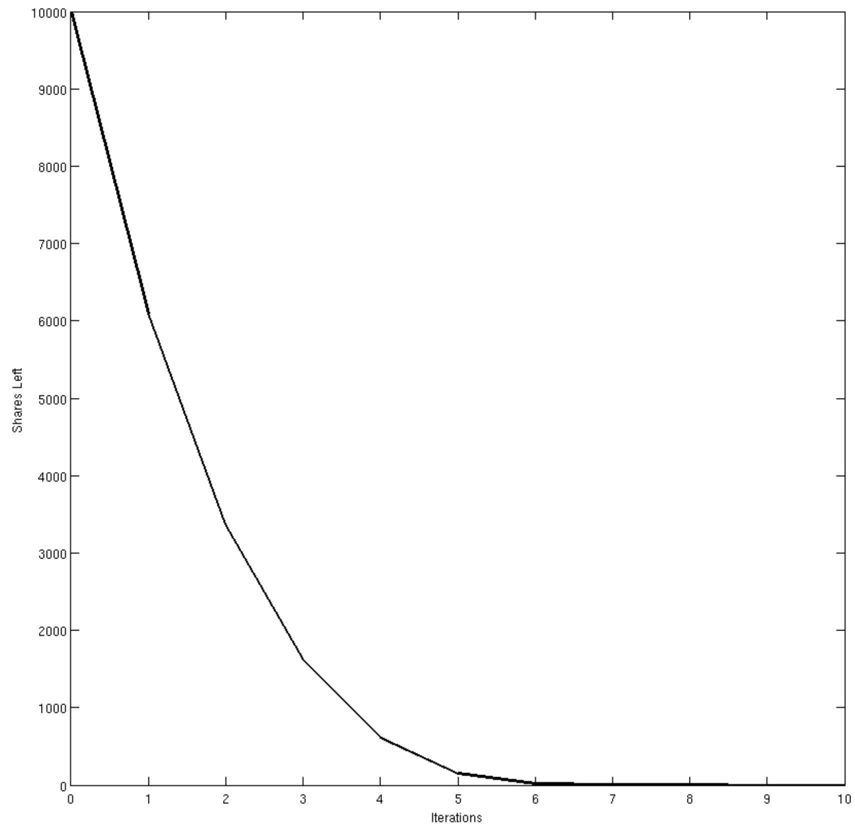
Portfolio Liquidation (Lukoil, 10000 stocks, N=10, λ=1)



Portfolio Liquidation (Lukoil, 10000 stocks, N=10, λ=10)



Portfolio Liquidation
(Lukoil, 10000 stocks, N=10, λ=1000)



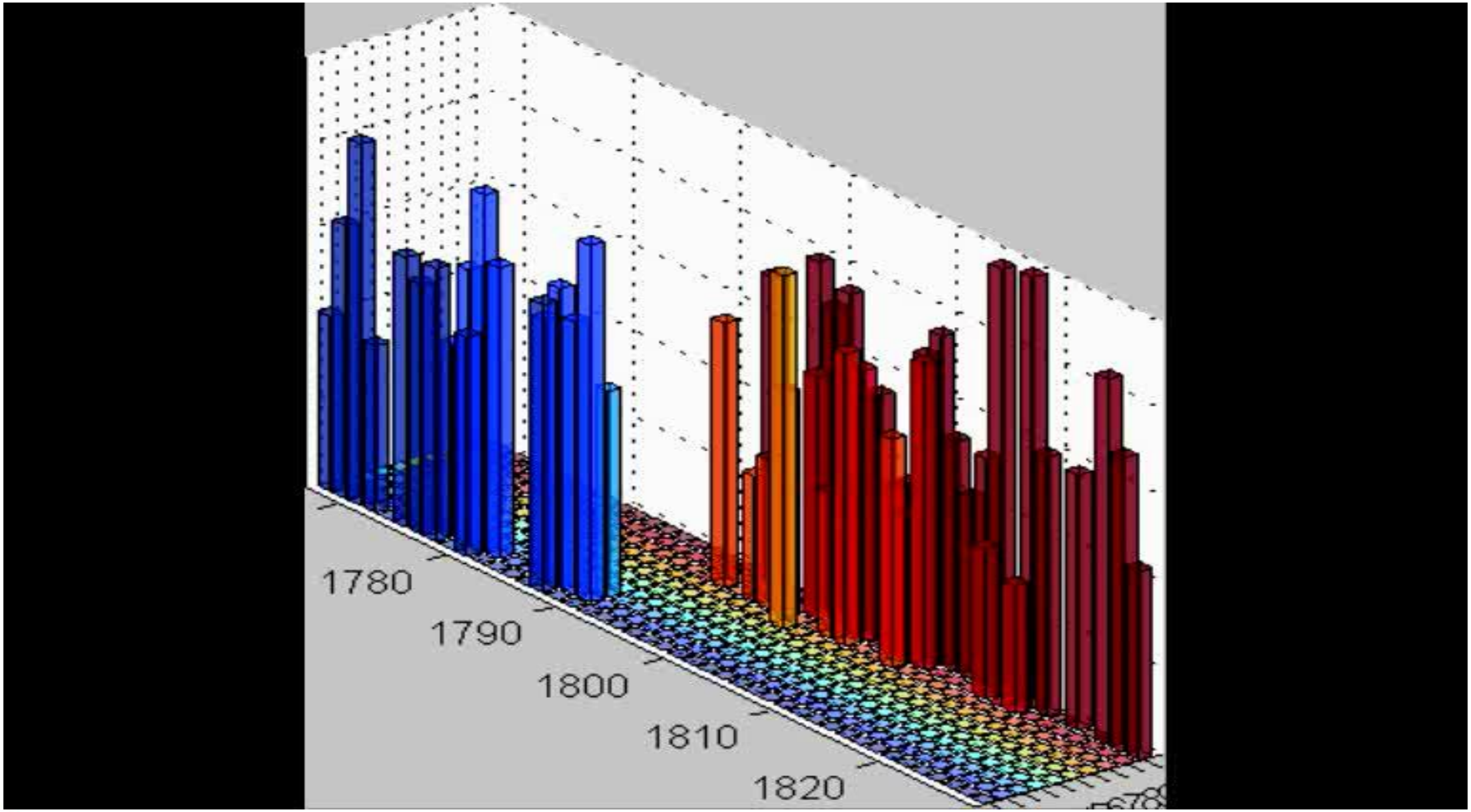
Measuring the Resiliency (Large, 2007)

- Resiliency is formalized in terms of a time-frame and probability of order book replenishment.
- Resiliency is captured simply by the way large trades (liquidity demand shock) alter the future intensities of fresh limit order submissions (resiliency events).
- LOB is modeled as mutually-exciting ten-variate Hawkes point process.

Categorization of Events (Biais, 1995)

№	Submission or Cancellation?	Buy or Sell?	Immediate Execution?	Moves prices?	Named
1	S	B	Yes (MO)	Yes	Market buy that moves the ask
2	S	S	Yes (MO)	Yes	Market sale that moves the bid
3	S	B	No (LO)	Yes	Bid between the quotes
4	S	S	No (LO)	Yes	Ask between the quotes
5	S	B	No (MO)	No	Market buy that doesn't move the ask
6	S	S	No(MO)	No	Market sale that doesn't move the bid
7	S	B	No (LO)	No	Bid at or below best bid
8	S	S	No (LO)	No	Ask at or above best ask
9	C	B	No	No	Cancelled bids
10	C	S	No	No	Cancelled asks

Trading Process



THANK YOU FOR YOUR ATTENTION!!!