Hedging with Futures: Dynamic Conditional Correlation Multivariate GARCH

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Outline

Motivation

- **Motivation**
  - There is no univocal evidence as to the effectivness of multivariate volatility models in estimating hedge ratious hedging with index future contracts
- **The goals of the research**
  - To apply multivariate conditional correlation GARCH models for estimation of dynamic hedge ratious
  - To provide an empirical comparison of different multivariate conditional correlation GARCH models
  - To measure the hedging effectiveness across differen markets
- **The Data**
  - Daily values of RTS, DAX, S&P500 and NASDAQ COMPOSITE indices and corresponding futures contracts from 1.01.2008 to 22.07.2010

The Hedging Strategy

- **Notations**
  - s_t and f_t are the natural logarithms of spot and future prices at time t
  - $r_t^s = s_t - s_{t-1}$ and $r_t^f = f_t - f_{t-1}$ are the returns on spot and future markets
  - $\Im_t$ - all the avaliable inormation up to time t
- **The Simple Dynamic Hedging Strategy**
  - The approximate return of a headged position

$$r_t^h = r_t^s - h_t \cdot r_t^f \tag{1}$$

  - The optimal (minimum variance) dynamic hedge ratio

$$h_t^* = \frac{Cov(r_t^s, r_t^f | \Im_{t-1})}{V(r_t^f | \Im_{t-1})} \tag{2}$$

The Bivariate GARCH Models

- **The observed process**

- The returns process $r_t = (r_t^s, r_t^f)'$

$$r_t = E(r_t|\Im_{t-1}) + \epsilon_t \tag{3}$$

$$\epsilon_t = \Sigma_{t-1}^{1/2} z_t \tag{4}$$

- z_t is a white noise process
- **The Conditional variance of the return process**

$$V(r_t|\Im_{t-1}) = \Sigma_t^{1/2} V(z_t)(\Sigma_t^{1/2})' = \Sigma_t. \tag{5}$$

$$\Sigma_t = D_t R_t D_t \tag{6}$$

  - R_t - the conditional correlation matrix
  - $D_t = diag(\sigma_{iit})$, $i = 1, 2$, σ_{iit} follow (different) univariate GARCH processes

The Conditional Correlations

- **Constant Conditional Correlation (CCC)**

$$R_t = R = (\rho_{ij}) \tag{7}$$

- **Dynamic Conditional Correlation (DCC)**

$$R_t = (diag(Q_t))^{\frac{1}{2}} Q_t (diag(Q_t))^{\frac{1}{2}} \tag{8}$$

$$Q_t = (1 - \theta_1 - \theta_2)\bar{Q} + \theta_1 u_{t-1} u'_{t-1} + \theta_2 Q_{t-1} \tag{9}$$

with

- $u_t = (u_{1t}, u_{2t})'$, $u_{it} = \frac{\epsilon_{it}}{\sqrt{\sigma^2_{iit}}}$ - standartizied errors, $i = 1, 2$
- $\bar{Q} = E(u_t u'_t)$

The Conditional Correlations

- **Asymmetric Dynamic Conditional Correlation (ADCC)**

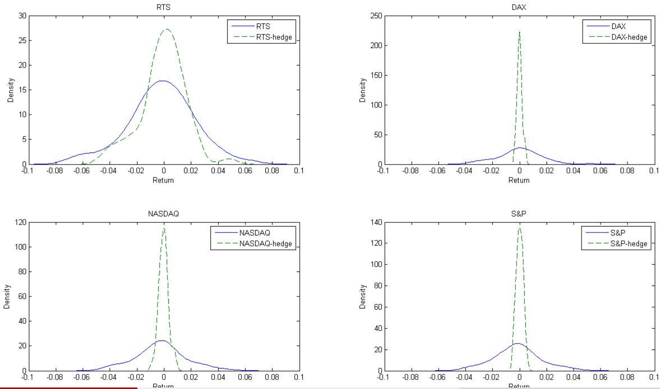
$$Q_t = (1-\theta_1-\theta_2)\bar{Q} - \theta_3\bar{N} + \theta_1 u_{t-1}u'_{t-1} + \theta_2 Q_{t-1} + \theta_3\eta_{t-1}\eta'_{t-1} \quad (10)$$

with

- $\eta_t = I[u_t < 0] \odot u_t$,
- $\bar{N} = E(\eta_t \eta'_t)$
- $I[u_t < 0]$ - indicator function
- $\odot$ - element by element multiplication

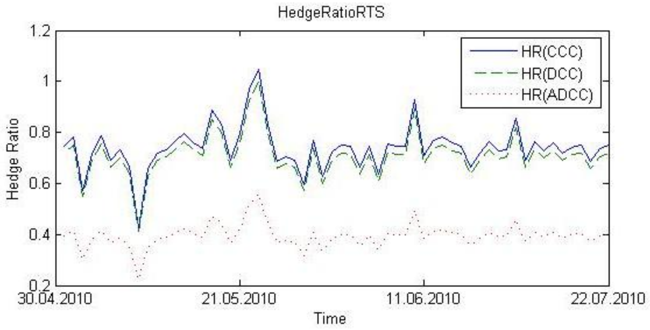
Main Result: Hedging Performance

- Data: RTS, DAX, S&P500 and NASDAQ500 COMPOSITE indexes and corresponding futures contracts
- Estimated by kernel smoothing method densities of hedged and unheadged returns



The Estimated Dynamic Hedge Ratio

- The series of estimated hedge ratios based on CCC, DCC and ADCC models for Russian RTS index



Summary

- Dynamic Conditional Correlation models, DCC and ADCC, unable to significantly improve hedging performance comparing to simplier Constant Conditional Correlation model
  - The lack of additional effectiveness for DCC and ADCC models can be explained by accumulating of the estimation errors due to larger number of the model's parameters and (or) potensial misspesification of univariate GARCHs
- There is a strong evidence for higher effectiveness of the dynamic hedging strategy applied to DAX and S&P500 indexes in comparison to NASDAQ and especially RTS indexes
  - The weak headging performance for RTS index points out the weaker effectiveness of Russian financial market
- Asymmetric Dynamic Conditional Correlation model tend to underestimate the optimal hedge ratios