NATIONAL RESEARCH UNIVERSITY

Consistency of government bond and sovereign CDS quotes: cross-market and cross-issuer analysis

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Outline

1. Motivation and Background
2. Methodology
3. Empirical Results and Interpretations
4. Conclusions

1. MOTIVATION & BACKGROUND

EFFAS-EBC Methodology (2006)

The EFFAS-EBC methodology (2006) was developed in order to construct risk-free zero-coupon yield curve and spreads in the euro zone using bonds of all major sovereign issuers in Eurozone.

The methodology was based on the best practice method of credit spreads calculation, described by van Deventer et al. (2013) as a ‘model independent’ procedure for calculation of credit spreads over US Treasury yield curve for creditworthy issuers.

Assuming a flat term structure of continuously compounded credit spread, the EFFAS-EBC methodology sets the risk-free zero-coupon yield curve and country-specific flat credit spreads so that issuer’s bond values calculated using them get best fit to the market prices (with specified degree of precision).

Being invariant to the choice of fitting method, this methodology is model-independent if concerning only the determination of the shape of risk-free yield curve.

The methodology produces “index-like” estimate of the risk-free yield curve.

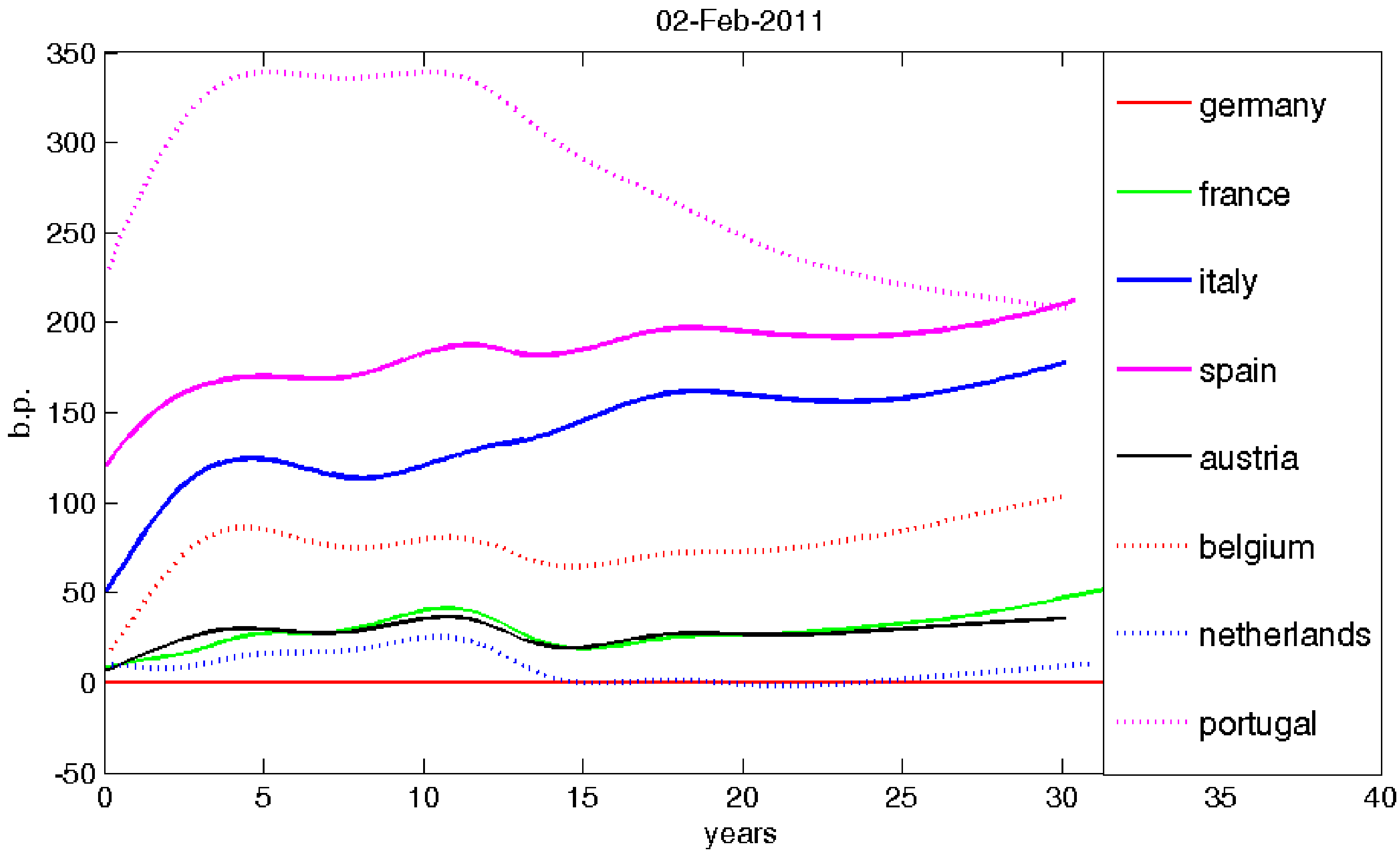
EFFAS-EBC Methodology (2006) (cont'd)

Although the EFFAS-EBC methodology performed well prior to the European debt crisis, the crisis changed the market picture dramatically, so results produced by the methodology are not consistent with market observations any more.

- Credit quality of several major sovereign issuers in Eurozone deteriorated significantly, so term structure of their spreads became far from flat. A well-known stylized fact, that the spreads of the low-grade bonds tend to be wider at the short end of the curve than at the long one.

- The assumption about the flatness of credit spread term structure after the crisis is no longer acceptable even for issuers of the highest credit quality.

Term Structure of Sovereign Spreads During the Eurozone Debt Crisis



The spreads are calculated over the German bond yield curve. The country-specific yield curves are fitted by non-parametric method.

Using CDS for Constructing Risk-Free Yield Curve

An additional information on a term structure of credit spreads is required in order to accurately determine the shape and the level of the risk-free zero-coupon curve in Eurozone.

The question arises: can the use of credit default swap (CDS) quotes (together with bond quotes) help to attain full term structure of risk-free interest rates?

Although the credit default swaps (CDS) are widely used to infer the credit risk priced in the bond credit spread, we understand that CDS spreads and bond yield spreads are not the same, so one should use CDS with a caution.

CDS vs Bonds

1. Technically, bond and CDS spreads are not the same, since bond spread has a meaning of excessive yield, whereas CDS spread has a meaning of excessive coupon. For fixed coupon bonds, even theoretically, the parity between CDS and bond spread holds only approximately and for the par traded bonds (Hull and White, 2000).

2. Blanco et al. (2005), Houweling and Vorst (2005), Zhu (2006) and others empiricaly show that there is a permanent negative basis (called CDS-bonds basis). For different Eurozone sovereign issuers the CDS-bond basis may be either negative or positive during Eurozone debt crisis (O'Kane, 2012). An amplitude of CDS-bond basis also depends on the choice of reference yield curve (proxy of risk-free rate).

3. Houweling and Vorst (2005), Longstaff et al. (2005) and others study bond and CDS prices within the paradigm of the risk-neutral pricing. They find that the model-based approach outperforms the simple estimation of CDS spreads with the bond spreads, but the estimates are still biased for any proxy of risk-free yield curve.

CDS vs Bonds (cont'd)

4. The CDS market typically leads the bond market in the price discovery process, but this effect is more pronounced in the US market than in Europe. For sovereign issuers the effect may be opposite in some cases. See Palladini and Portes (2011) or Fontana and Scheicher (2011).

5. Different factors may impact bond and CDS prices:
   - Different liquidity premiums;
   - Convenience yield (bonds);
   - FVA, CVA, DVA (CDS).

Eurozone Sovereign CDS

Sovereign issuers usually have many bonds of different maturities and the corresponding CDS are actively traded for different tenors. At least, from the end of 2009, when the trading activity in the sovereign CDS market has increased dramatically (see International Monetary Fund Global Financial Stability Report, 2013) and to the end of 2011, when the ban on “naked” positions in EU sovereign CDS contracts was announced (see report Research note on EU uncovered sovereign CDS ban by ISDA, 2014).

However trading activity is mainly concentrated in the segment of USD-denominated sovereign CDS. See Chen et al. (2011) and O’Kane (2012). The euro-denominated CDS are not actively traded and price information on such CDS is usually based on quotes of the market dealers.

The spreads of USD CDS on EUR sovereign bonds have a basis over corresponding EUR CDS spreads, since USD CDSs protect against EUR devaluation risk. So the adjustment of USD CDS spreads requires a special model, which is non-trivial to build. See for example Ehlers and Schoenbucher (2006).

“Index-Like” Risk-Free Curve

In order to develop a tractable model/methodology one has to make some simplifying assumptions and/or ignore some characteristics of bond/CDS markets, especially if those characteristics are not crucial for the particular purposes, although it makes estimates rougher. It is also important, especially for actuarial purposes, to obtain robust estimate of risk-free curve.

The “index-like” methodology of construction of risk-free curve may ensure the robustness of results, as it allows for averaging out idiosyncratic irregularities. However index itself is not enough. A proper selection procedure is needed to ensure homogeneity and consistency of data and to filter out the individual issuers or instruments significantly distorting the aggregated estimate due to some factors.

2. METODOLOGY

Approach principles

- Risk-neutral (no-arbitrage) pricing paradigm.
- Only euro-denominated instruments are used.
- Independence with respect to the choice of fitting method.
- Keeping approach tractable.
- “Index-like” estimate of risk-free yield curve.
- Balance of accuracy and smoothness of risk-free curve estimation.

Interest Rate Modelling

- Discount function:

$$D_t(s) = \exp\left[-s \cdot r_t(s)\right] = \exp\left[-\int_0^s f_t(x)dx\right] = E_t \exp\left[-\int_t^{t+s} r_x dx\right]$$

- where $r_t = r_t(0) = f_t(0)$ is the default-free spot rate, $f_t(x)$ is the instantaneous forward rate prevailing at time t for the future time $t+x$, and $\boldsymbol{E}_t$ denotes the conditional expectation given available at time t information with respect to the risk-neutral measure.

Default Probability Modelling

- Survival probability function:

$$Q_t(s) = \exp\left[-s \cdot \Lambda_t(s)\right] = \exp\left[-\int_0^s \lambda_t(x)dx\right] = E_t \exp\left[-\int_t^{t+s} \lambda_x dx\right]$$

- where Λ_t is the hazard process, $\lambda_t(x)$ is the instantaneous hazard rate prevailing at time t for the future time $t+x$, and $\boldsymbol{E}_t$ denotes the conditional expectation given available at time t information with respect to the risk-neutral measure.

Pricing Formula

- The present value at time 0 of a defaultable instrument promising a cash flow F_i at time t_i and paying nothing at default can be then written as:

$$PV = \sum_{i=1}^{n} F_i D_0(t_i) Q_0(t_i) = \sum_{i=1}^{n} F_i \exp\left[-\int_0^{t_i} \{ f_0(x) + \lambda_0(x) \} dx \right]$$

- For a non-zero recovery rate this formula needs a slight modification to allow for the repayment of the recovered value at the (random) time of the default.

Bond and CDS Pricing Formulas

- For a bond with the face value of one, the annual coupon *c* paid at times t_i and constant loss given default L^{bond}, defined according to the loss of face value convention, and assuming constant recovery, we obtain the following formula.

$$P^{bond} = c\sum_{i=1}^{n}(t_i - t_{i-1})D(t_i)Q(t_i) + 1\cdot D(t_n)Q(t_n) + \int_0^{t_n}\left[(1-L^{bond})D(x)\right]d(1-Q(x))$$

- For CDS pricing, we use the same type of formula, taking into account that only a fraction of the CDS premium, denoted by L^{CDS} , is to be paid for the period in which the credit event occurs. *S* is the standard CDS premium and *I(x)* is the (random) index of the last premium payment before the time *x.*

$$P^{CDS} = \int_0^{t_n} L^{CDS} D(x)d(1-Q(x)) - S\left[\sum_{i=1}^{n} D(t_i)(t_i - t_{i-1})Q(t_i) + \int_0^{t_n} D(x)(t_{I(x)} - x)d\left(1-Q(x)\right)\right]$$

Risk-free yield curve estimation

- We measure the quality of price replication for them by normalising the pricing error by the quoted bid-ask spread (liquidity cost measure).

- Let the dirty bid and ask quotes of this bond be $a_{j,k}$ and $b_{j,k}$ respectively for j^{th} of the k^{th} issuer, and suppose there are n_k bonds of the k^{th} issuer. The goodness-of-fit measure of theoretical bond prices:

$$J_k^{bond} = \frac{1}{n_k} \sum_{j=1}^{n_k} \left[\frac{1}{a_{j,k} - b_{j,k}} \left(P_{j,k}^{bond} - \frac{a_{j,k} + b_{j,k}}{2} \right) \right]^2$$

- Let the quoted bid and ask upfront payments for the CDS *j* on the issuer *k* be $A_{j,k}$ and $B_{j,k}$ respectively with the total number of CDS N_k. Thus, we can choose the goodness-of-fit measure for the CDS theoretical prices in a form:

$$J_k^{CDS} = \frac{1}{N_k} \sum_{j=1}^{N_k} \left[\frac{1}{A_{j,k} - B_{j,k}} \left(P_{j,k}^{CDS} - \frac{A_{j,k} + B_{j,k}}{2} \right) \right]^2$$

Risk-Free Yield Curve Estimation

- In order to estimate the term structure of the risk-free rates from single issuer data, one might fix *k* and minimize some convex combination of J_k^{bond} k and J_k^{CDS}. In order to construct an index-like estimate we have to use all J_k in the goodness-of *t* measure.

- If the unknown functions f() and $\lambda_k()$ belong to any parametric class, then overall goodness-of-fit measure is simply:

$$\sum_{k=1}^{N} J_k^{bond} + \sum_{k=1}^{N} J_k^{CDS} \to \min$$

- For non-parametric fitting the problem is ill-posed, so suitable regularisation is needed to provide robustness. Goodness-of-fit measure is modified:

$$\frac{1}{N}\sum_{k=1}^{N}\left[J_k^{bond}\right] + \alpha w^{bond}(f) + \frac{1}{\tilde{N}}\sum_{k=1}^{N}\left[J_k^{CDS} + \beta_k w_k^{CDS}(\lambda_k)\right] \to \min_{f(\cdot),\lambda_k(\cdot)}$$

- Where w^{bond}, w^{CDS} – regularisation terms and α, β_k – regularisation parameters.

Data Selection Principles

- In order to produce a fully-edged index-like risk-free interest rates in the euro zone, it would be necessary to examine all issuers and instruments available and establish some sensible selection criteria among all relevant issuers and instruments based on credit quality of issuers, liquidity of their bonds, their maturity, etc.

- The main purpose of such a selection is to filter out the individual issuers or instruments significantly distorting the aggregated estimate due to issuer-specific or instrument-specific factors.

- Some reasonable selection principles can be found in “A Guide to the EFFAS Standard Rules” by Brown (2002). Nevertheless, in our case these principles are insufficient, as we also consider CDS contracts.

- It is also important to ensure the consistency of data sources, especially for the aggregated data.

Data Description

- Main sovereign debt issuers in Eurozone: Germany, France, Italy, Spain, Portugal, Austria, Belgium, Netherlands.

- Bonds:
  - Fixed-coupon or zero-coupon bonds.
  - Euro-denominated
  - Maturity from 1 to 30 years (up to 40+ issues of particular country)
  - End-of-day (EOD) bid and ask quotes
  - Source: Markit (Iboxx Eur Benchmark Index Constituents)

- CDS:
  - Up-front + 100 bp. fixed coupon
  - Euro-denominated
  - Tenors: 6m, form 1 to 5 years, 7 year, 10 year, 20 year и 30 year.
  - Restructuring clauses: Modified-Modified.
  - End-of-day (EOD) up-front prices
  - Source: Thomson Reuters

Accuracy Measure

- The bid and ask quotes can be transformed into model-independent constraints (no-arbitrage bounds) for the bond yield curve, which the absence of arbitrage opportunities imposes on yield-curve estimates (Jaschke, 1998).
- If no-arbitrage bounds are empty, the bond quotes are inconsistent.
- If opposite, the bounds can be seen as an accuracy measure of the problem and as a criteria of acceptability for any estimate of the bond yield curve.

Accuracy Measure (cont'd)

- In our notations, the no-arbitrage bounds for the individual yield curve of some issuer *k* are the solutions of following optimization problem.

$$\begin{cases} r(t_i) = -\frac{1}{t_i} lnD(t_i) \\ D(t_i) \to \max, \min \\ 1 \geq D(t_i) \geq \ldots \geq D(t_i) \geq 0 \\ a_k \geq \sum_{i=1}^{N} F_{i,k} D(t_i) \geq b_k \end{cases}$$

- In our setting it is possible to extend this approach in order to construct the no-arbitrage bounds for the risk-free curve estimates using both bond and CDS quotes.

3. EMPIRICAL RESULTS AND INTEPRETATIONS

Bond-Implied No-Arbitrage Bounds

- In our data sample the no-arbitrage bounds for the country-specific bond yield curves are frequently empty. In this case it is hard to infer a reasonable accuracy indicator for the bond yield (risky) curve estimates.
- Some countries in our data sample have many bond issues with very tight bid-ask spreads and very close maturity dates. In such a case the consistency of the bond quote data became very sensitive to the small perturbation in quotes.

Bond-Implied No-Arbitrage Bounds (cont'd)

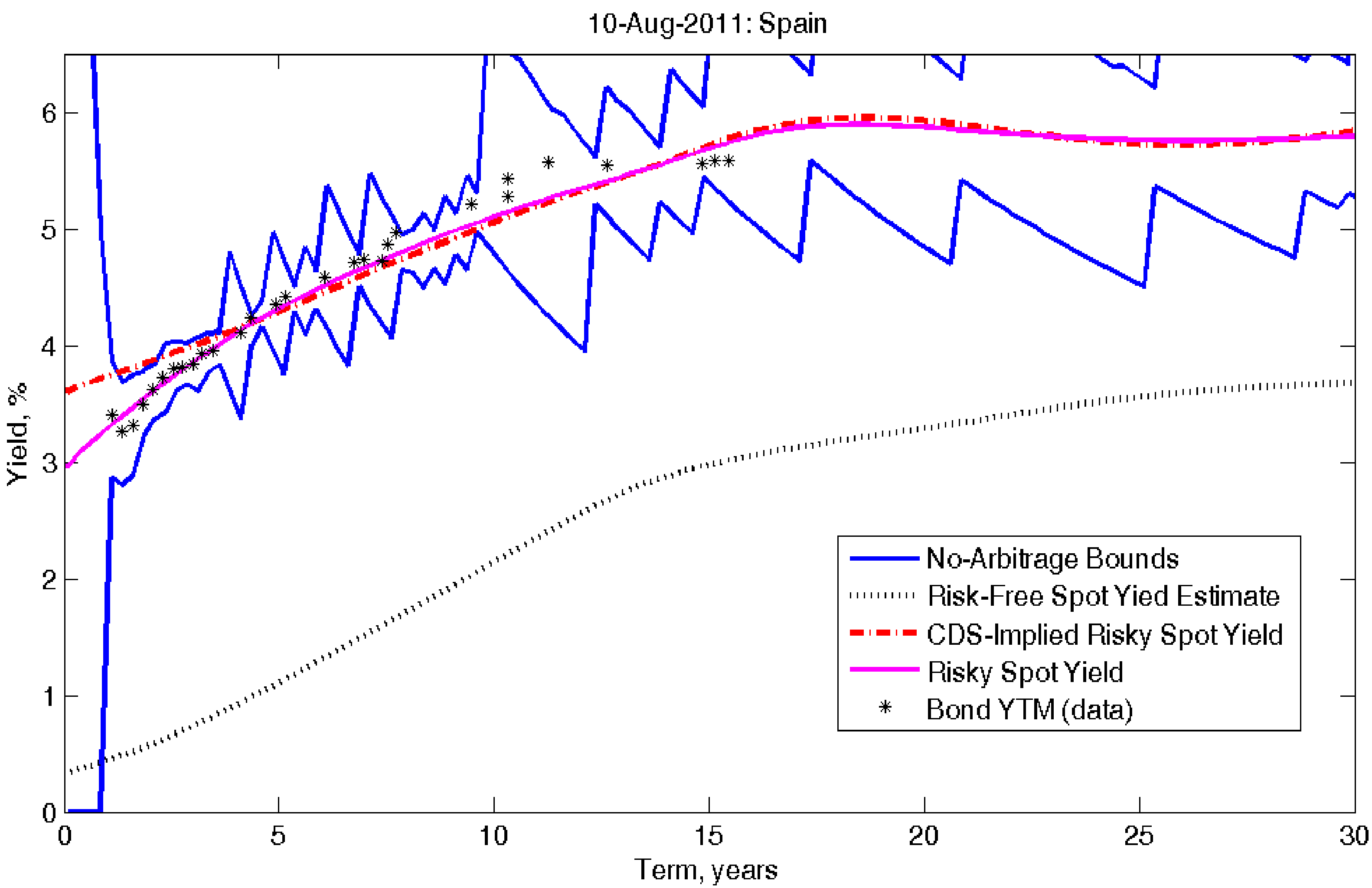
- The possible alternative to avoid this problem is to exclude some issues from consideration. For this purpose issues may be prioritized according some economic reason: amount outstanding, on-the-run/off-the-run, trading volumes etc. Therefore the benchmark bonds are regarded as the main candidates to be used in calculations.

- The following figures demonstrate the no-arbitrage bounds for Spain and Italy.

Bond-Implied No-Arbitrage Bounds (cont’d)

- On 10 August 2011 all Spanish bond quotes are consistent and the no-arbitrage bounds can be constructed for all Spanish issues.

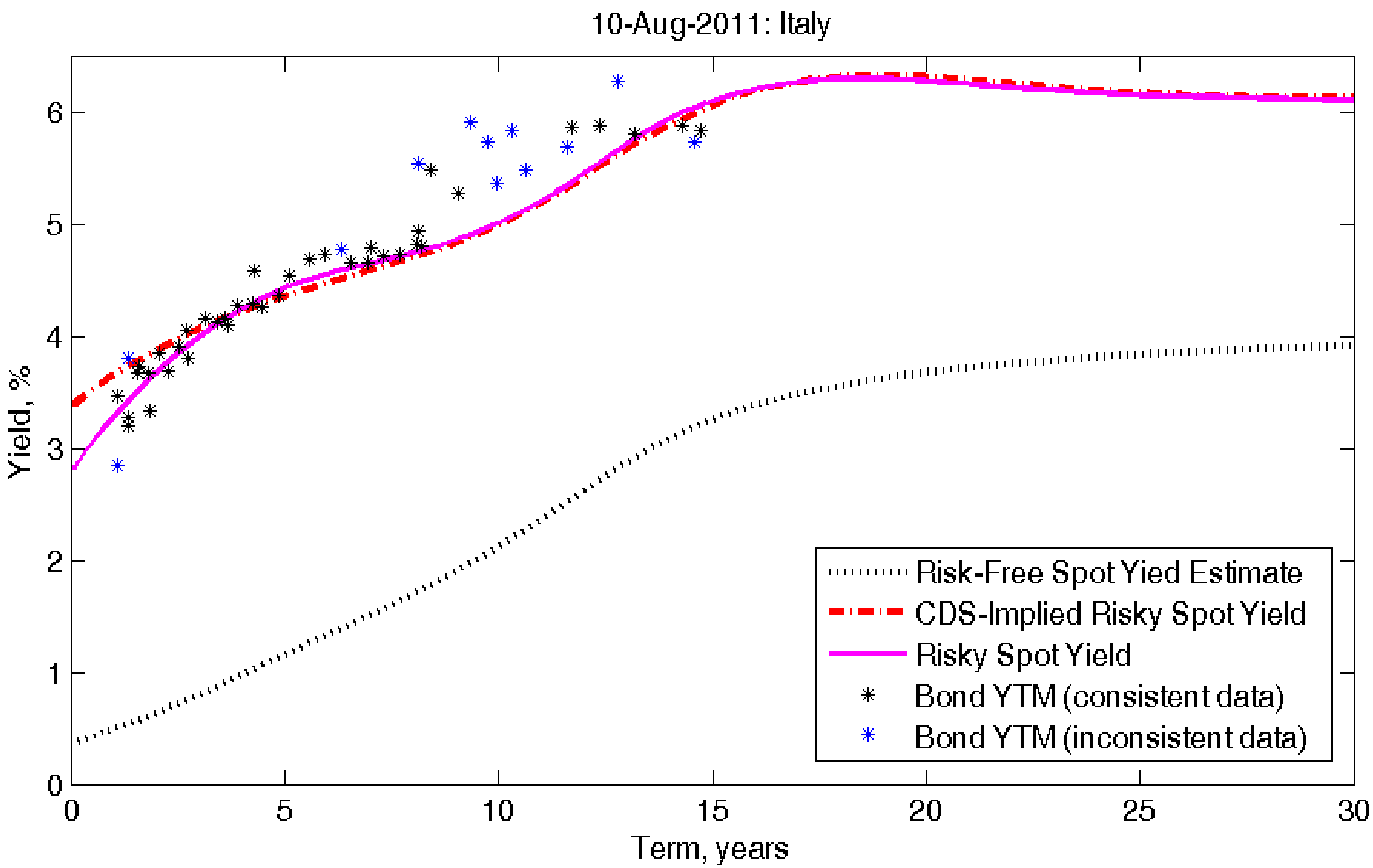
Bond-Implied No-Arbitrage Bounds (Spain)



Bond-Implied No-Arbitrage Bounds (cont'd)

- In contrast, some Italian bonds have to be excluded from the no-arbitrage bounds constructions process in order to make these bound non-empty.

Bond-Implied No-Arbitrage Bounds (Italy)



Iboxx Eur Benchmark Criteria

- The Iboxx Eur Benchmark Index criteria includes bond types, minimum time to maturity, amount outstanding and issuer's credit rating. See http://www.markit.com/assets/en/docs/fact-sheets/MKT_iBoxx_EUR_Benchmark_Indices_factsheet.pdf
- The constitutes of index are revised monthly and any issuer downgraded bellow investment grade (IG) has to be excluded from the index base in a following month.
- Greece lost IG rating by S&P in April 2010 and was excluded form index base in June 2010. In March 2012 the ISDA EMEA Determinations Committee declared the Greek debt restructuring credit event.
- Requesting for activation of financial aid mechanism, Portugal lost IG rating by Moody's at the beginning of July 2011 and was excluded from index base by the end of the same month.

Tightness factor

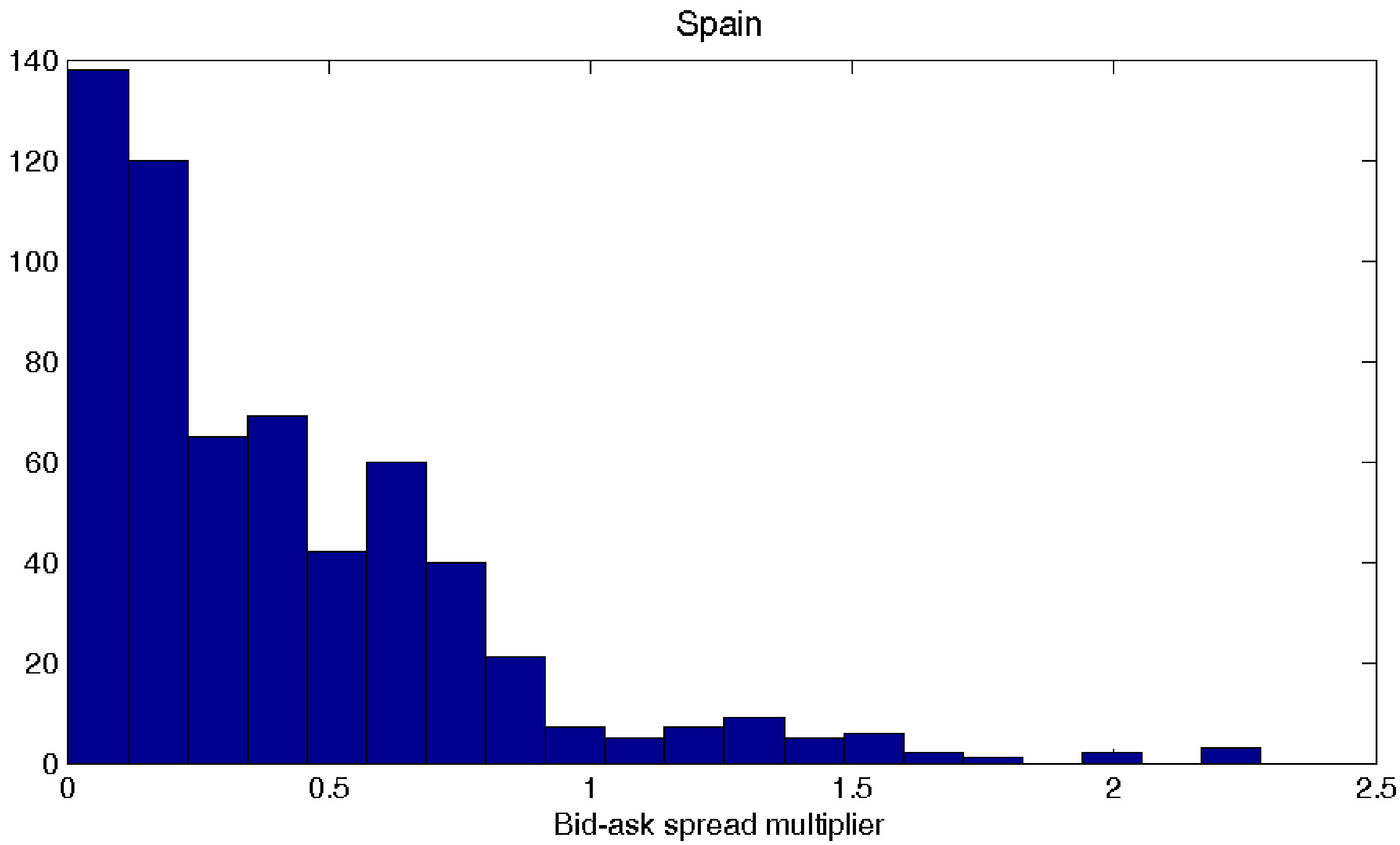
- Another alternative to ensure a non-empty no-arbitrage bounds is widening bid and ask quotes for all issuer, while preserving mid price, to the extent when bond quotes become consistent.

- We introduce the notion of the tightness factor. The market tightness factor is the least factor, by which one needs to widen the bid-ask spreads to make the problem.

Tightness factor (Spain)

- For the most of dates it is enough to widen bid-ask spread of Spanish bonds by 50% to ensure existence of no-arbitrage bounds. The result holds for most of the countries.

Distribution of the tightness factor (Spain)



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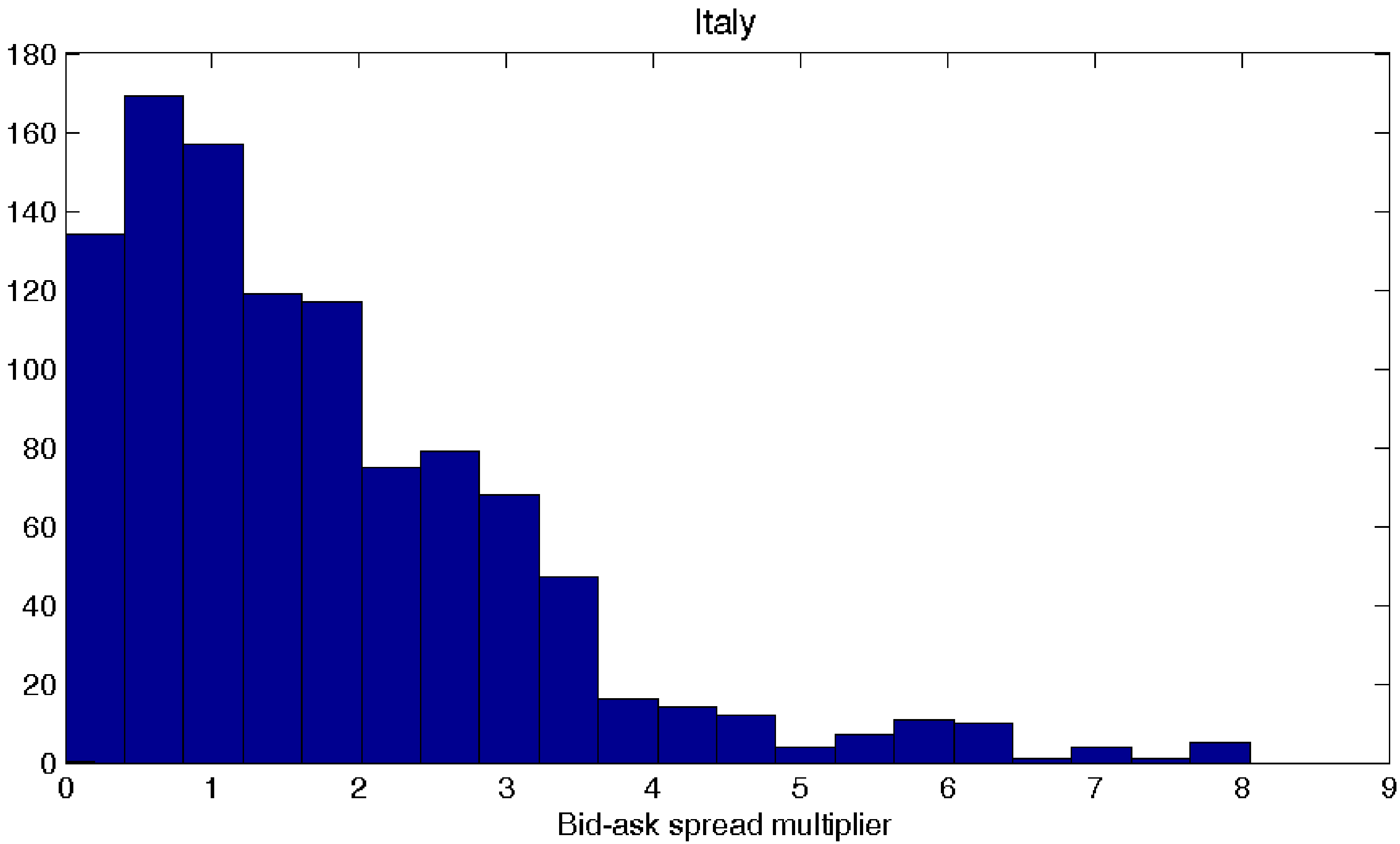
Tightness factor (Italy)

- In order to ensure non-empty no-arbitrage bounds for the most of the dates one should assume 4-times wider bid-ask spreads for all Italian bonds. The result also holds for Germany, but bid-ask spread for German bonds are much more narrow.

Distribution of the tightness factor (Italy)



No-arbitrage Bounds for Single-Issuer Risk-Free Yield Curve

Now we can state the optimization problem for the no-arbitrage bounds for the risk-free discount factor for every single issuer.

$$
\begin{cases}
D(t_i) \to \min, \max, \\
b_{j,k} \le P_{j,k}^{bond} \le a_{j,k}, j = 1..n_k, \\
B_{j,k} \le P_{j,k}^{CDS} \le A_{j,k}, j = 1..N_k, \\
D(0) = 1, D(t) \ge 0, D(t) \text{ decreases}, Q_k(0) = 1, Q_k(t) \ge 0, Q_k(t) \text{ decreases},
\end{cases}
$$

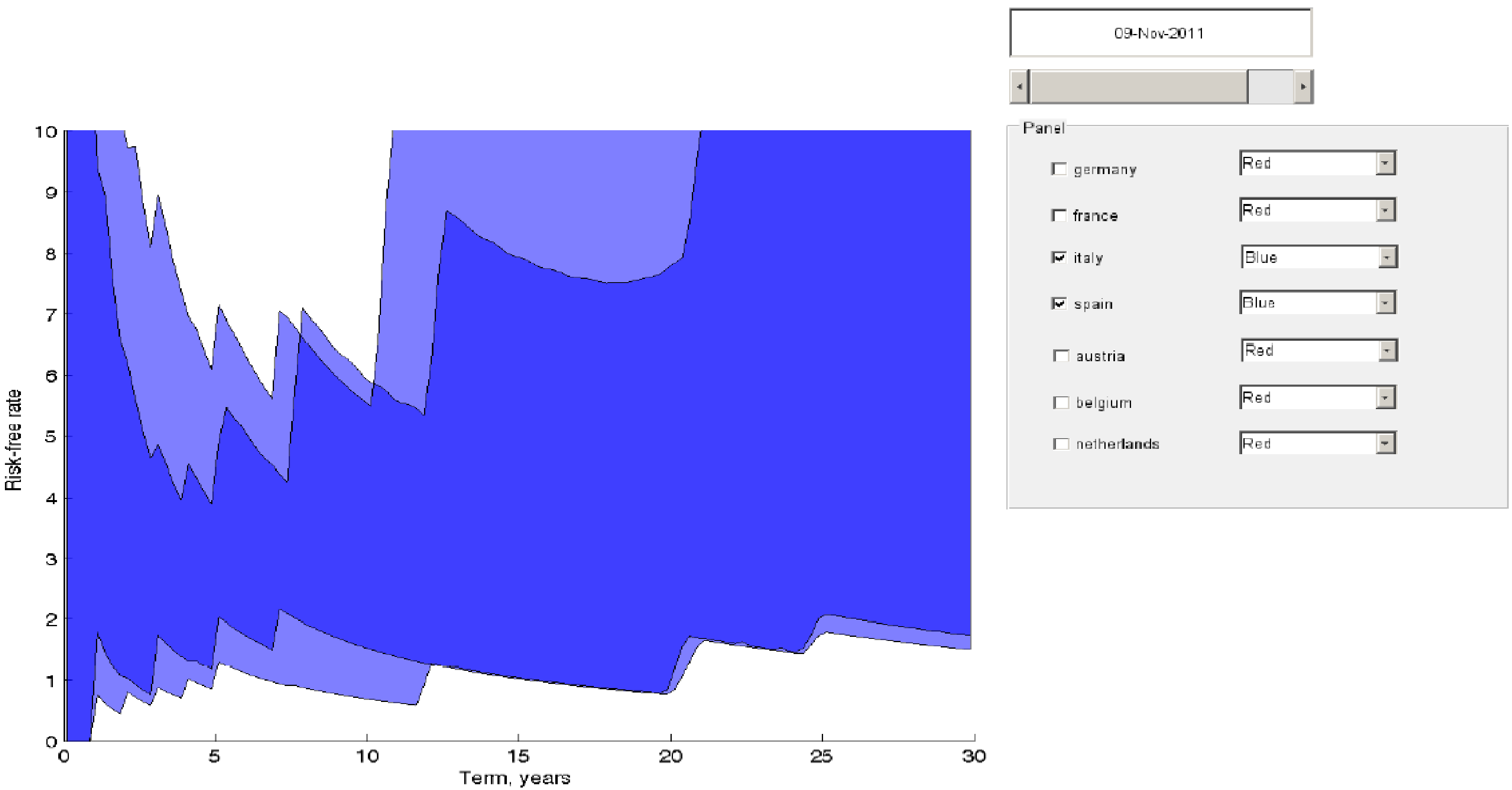
It turns out that these bounds are always non-empty, proving that the bonds and CDS are consistent with each other on a single-issuer level.

Accuracy of Estimation of Risk-Free Yield Curve

- As was mentioned above, our methodology allows us to construct no-arbitrage bounds for risk-free yield curve in Eurozone using all sovereign bond and CDS quotes.

- However we find that no-arbitrage bounds for the index risk-free yield curve are empty, even if respective bounds implied by the single issuer's bonds and CDS can be constructed, that means inconsistency of bond and CDS quotes of different issuers.

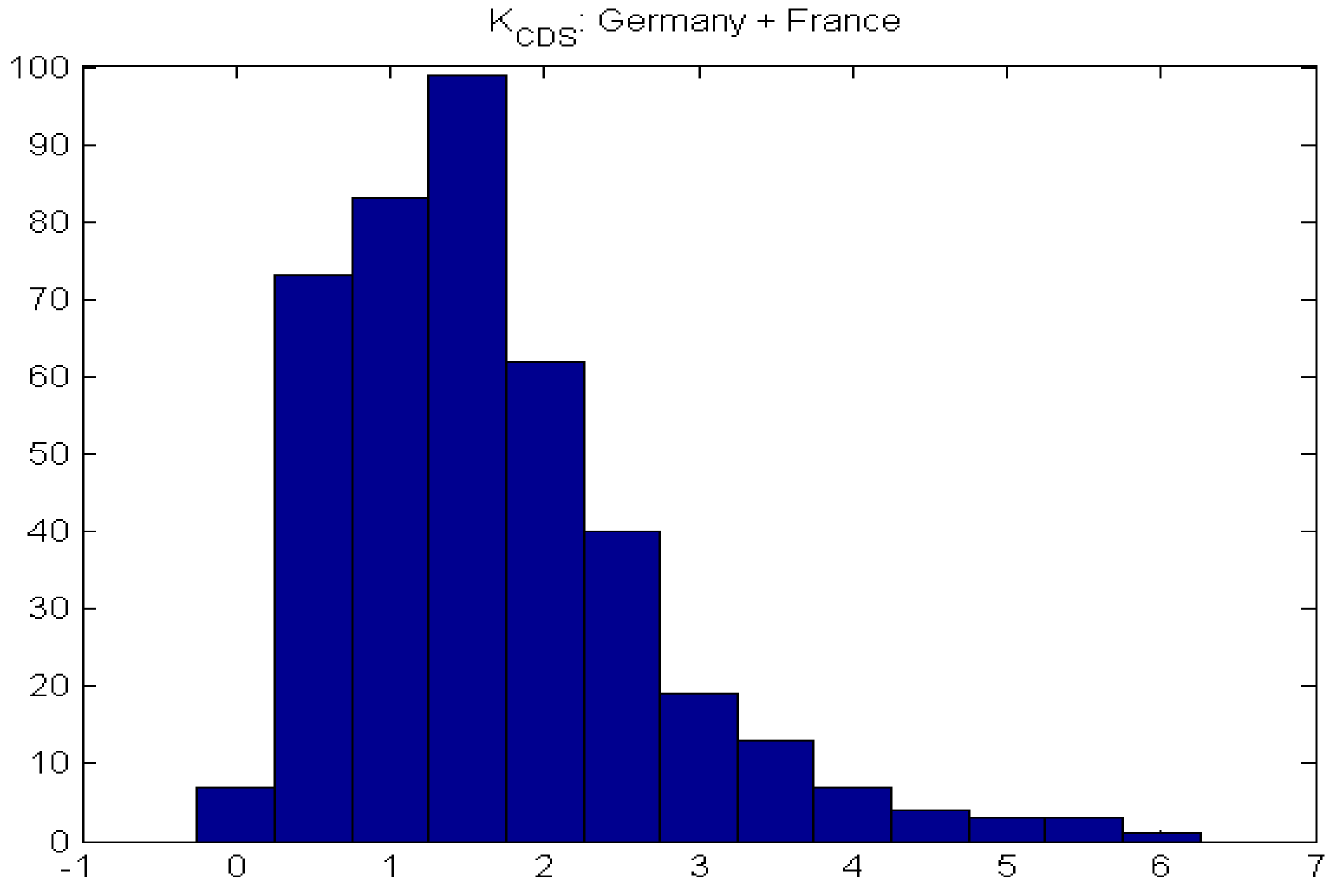
Accuracy of Estimation of Risk-Free Yield Curve (Italy, Spain)



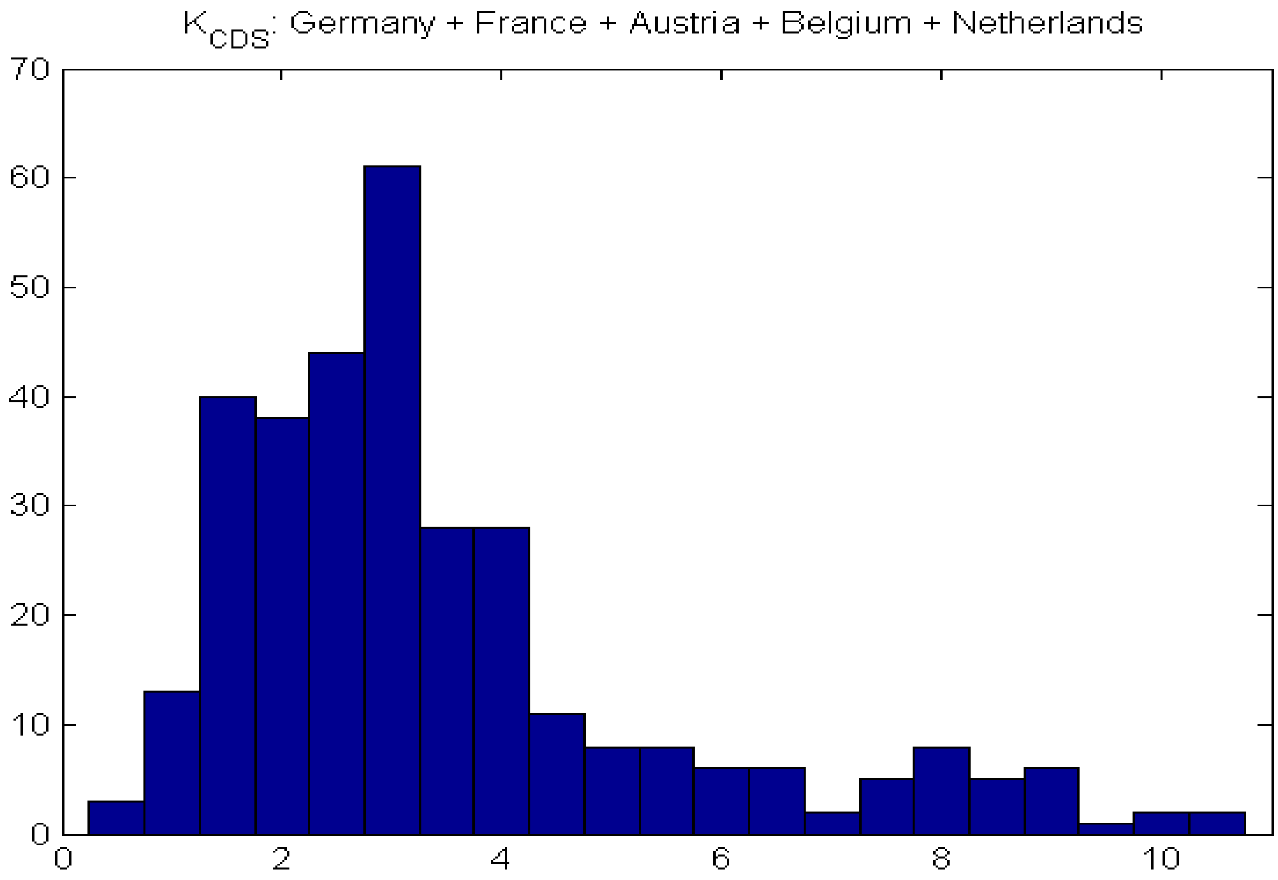
Accuracy of Estimation of Risk-Free Yield Curve (cont'd)

Issuer Group(s)	N of consistent days
Germany, France	116 (36%)
Italy, Spain	339 (77%)
Netherlands, Austria, Belgium	176 (54%)
Germany, France, Netherlands, Austria, Belgium	7 (2%)
Germany, France, Italy, Spain	0 (0%)
Germany, France, Netherlands, Austria, Belgium, Italy, Spain	0 (0%)

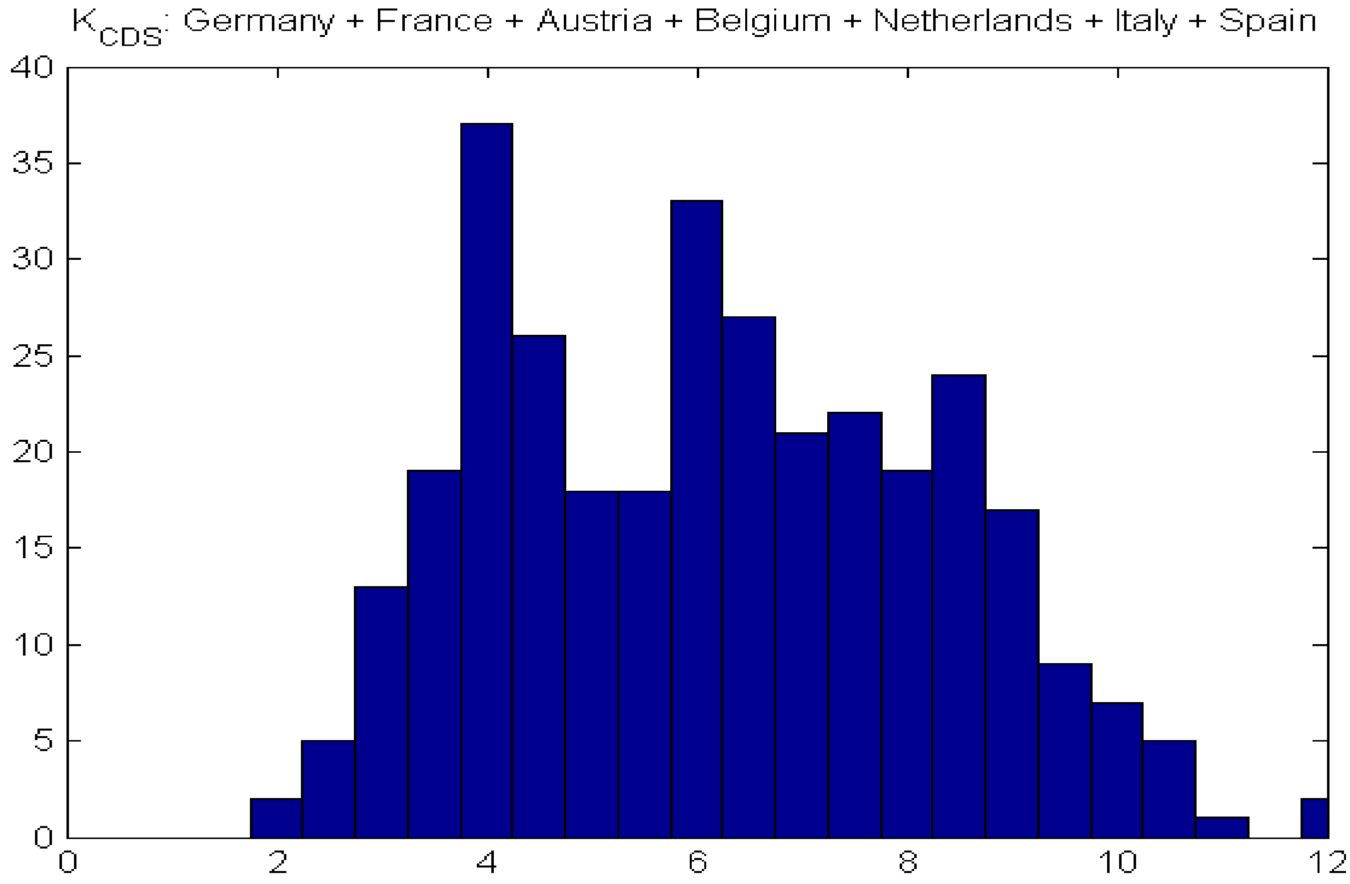
Distribution of the tightness factor



Distribution of the tightness factor



Distribution of the tightness factor



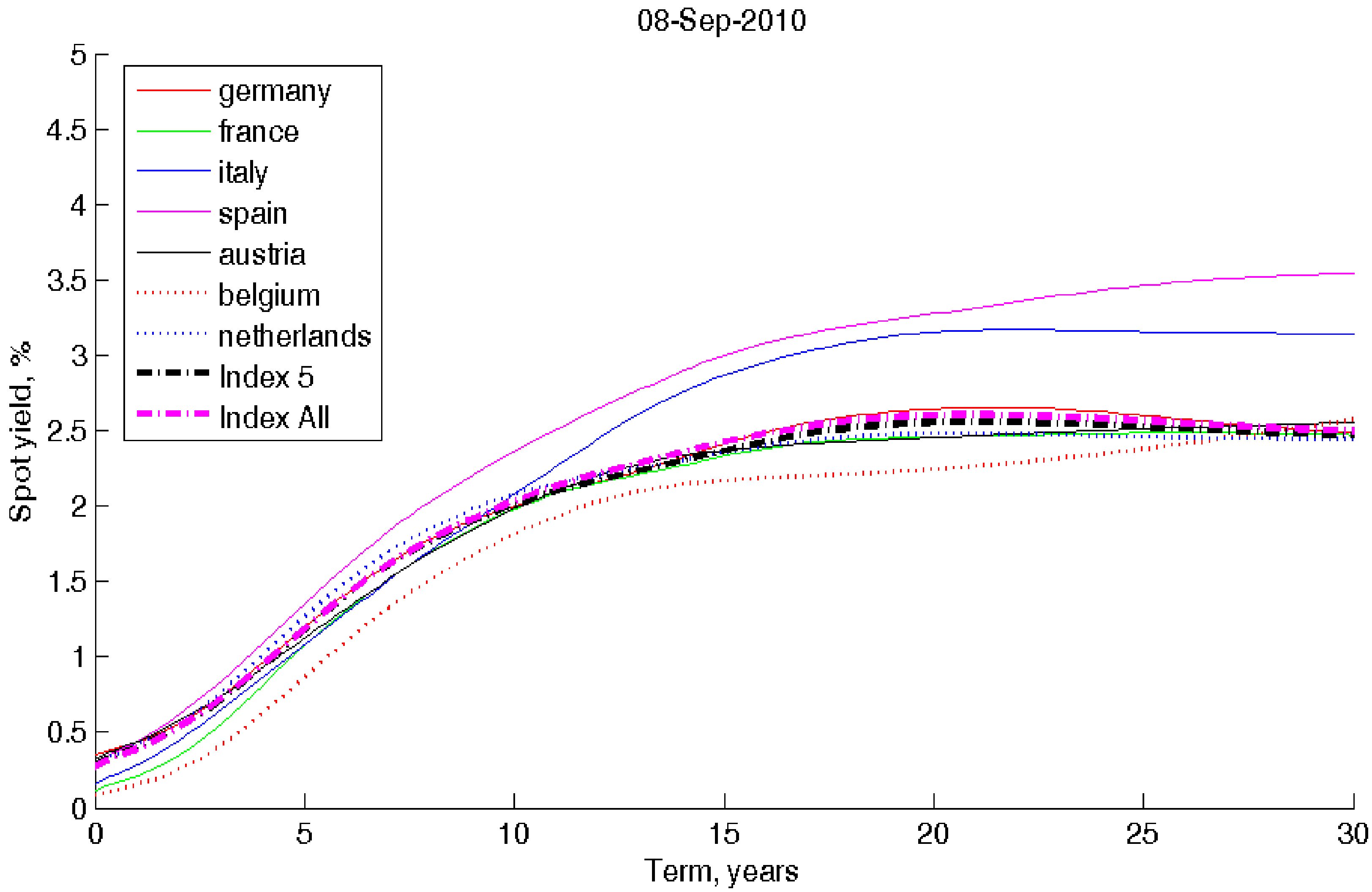
Dynamics of Country-Specific Estimates of Risk-Free Yield Curve

- The following slides demonstrate the weekly dynamics of country-specific estimates of Eurozone risk-free yield curve constructed applying the non-parametric fitting method to the bond and CDS data for particular sovereign issuer (Germany, France, Austria, Belgium, Netherlands, Italy, Spain) and two index estimates.
- The “Index 5” curve is constructed from the bonds and CDS of Germany, France, Austria, Belgium, Netherlands. The “Index All” estimate also takes into account Italy and Spain.
- We excluded the estimates based on Portuguese instruments, because they are significantly different from those shown bellow.

Dynamics of Country-Specific Estimates of Risk-Free Yield Curve (cont’d)

- At the beginning of data sample the risk-free rate curve estimates were fairly close to each other, at least for maturities up to 10 years.

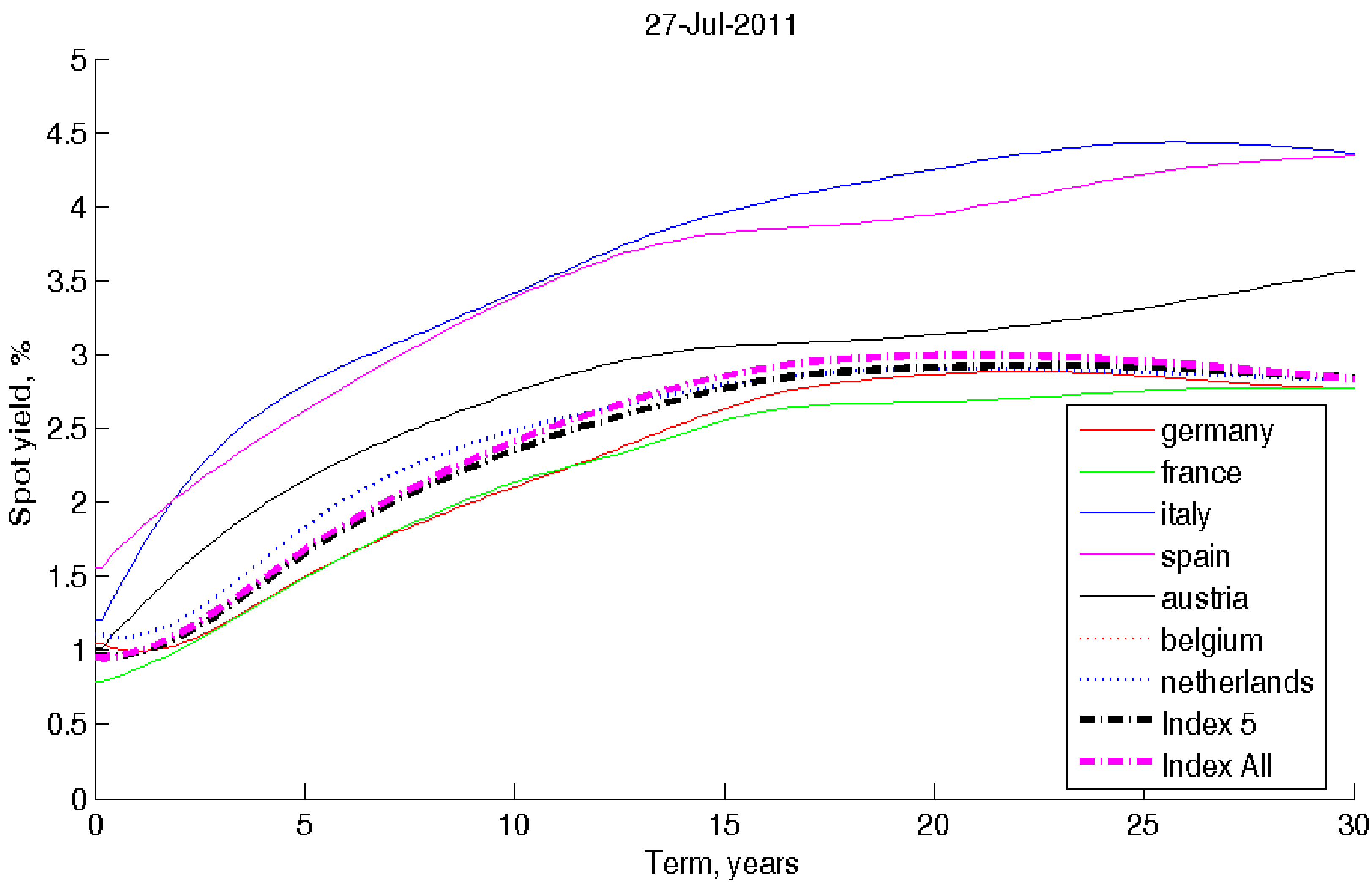
Dynamics of Country-Specific Estimates of Risk-Free Yield Curve (cont'd)



Dynamics of Country-Specific Estimates of Risk-Free Yield Curve (cont'd)

- By the end of July 2011 the country-specific estimates diverged significantly.

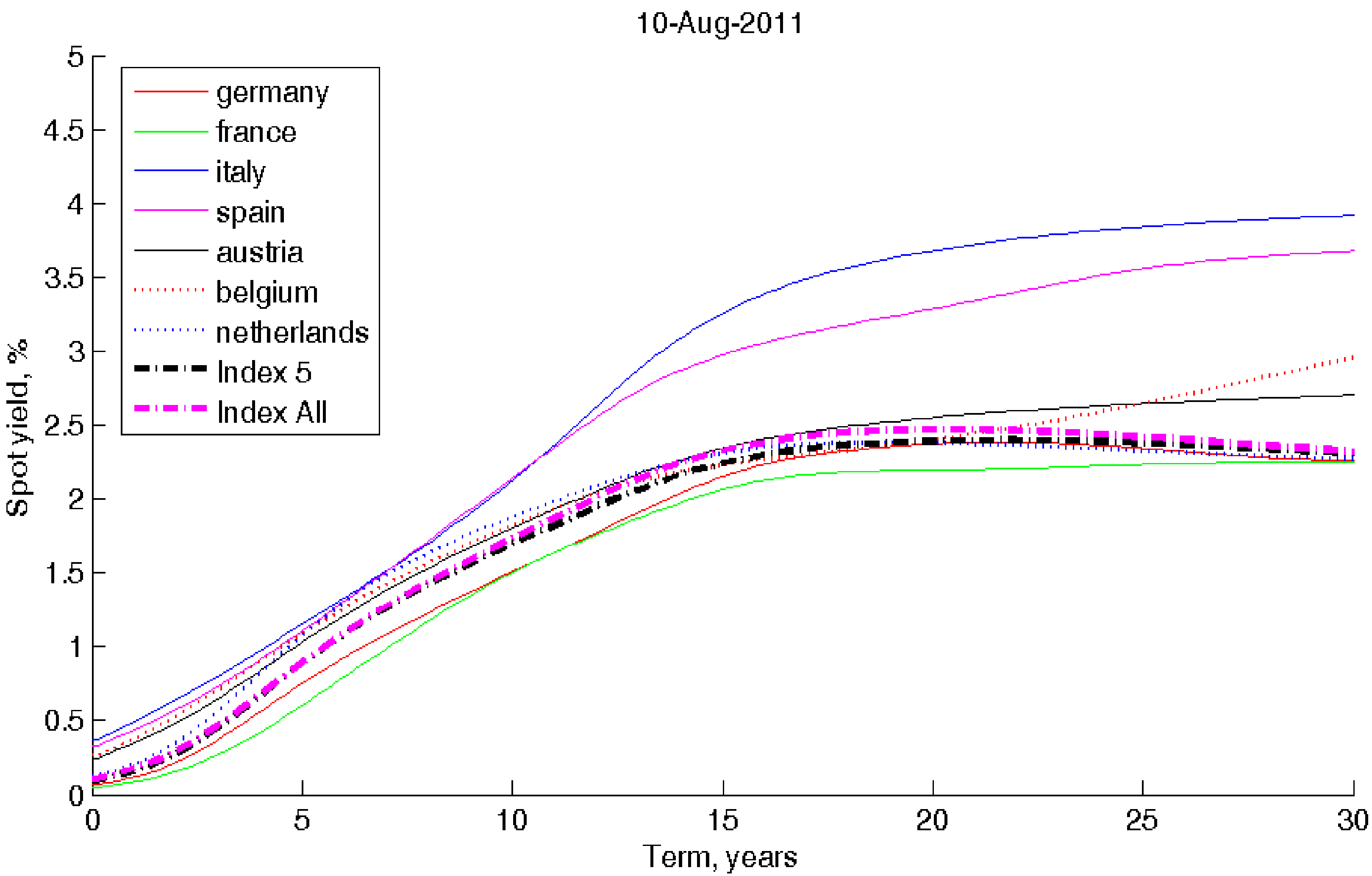
Dynamics of Country-Specific Estimates of Risk-Free Yield Curve (cont'd)



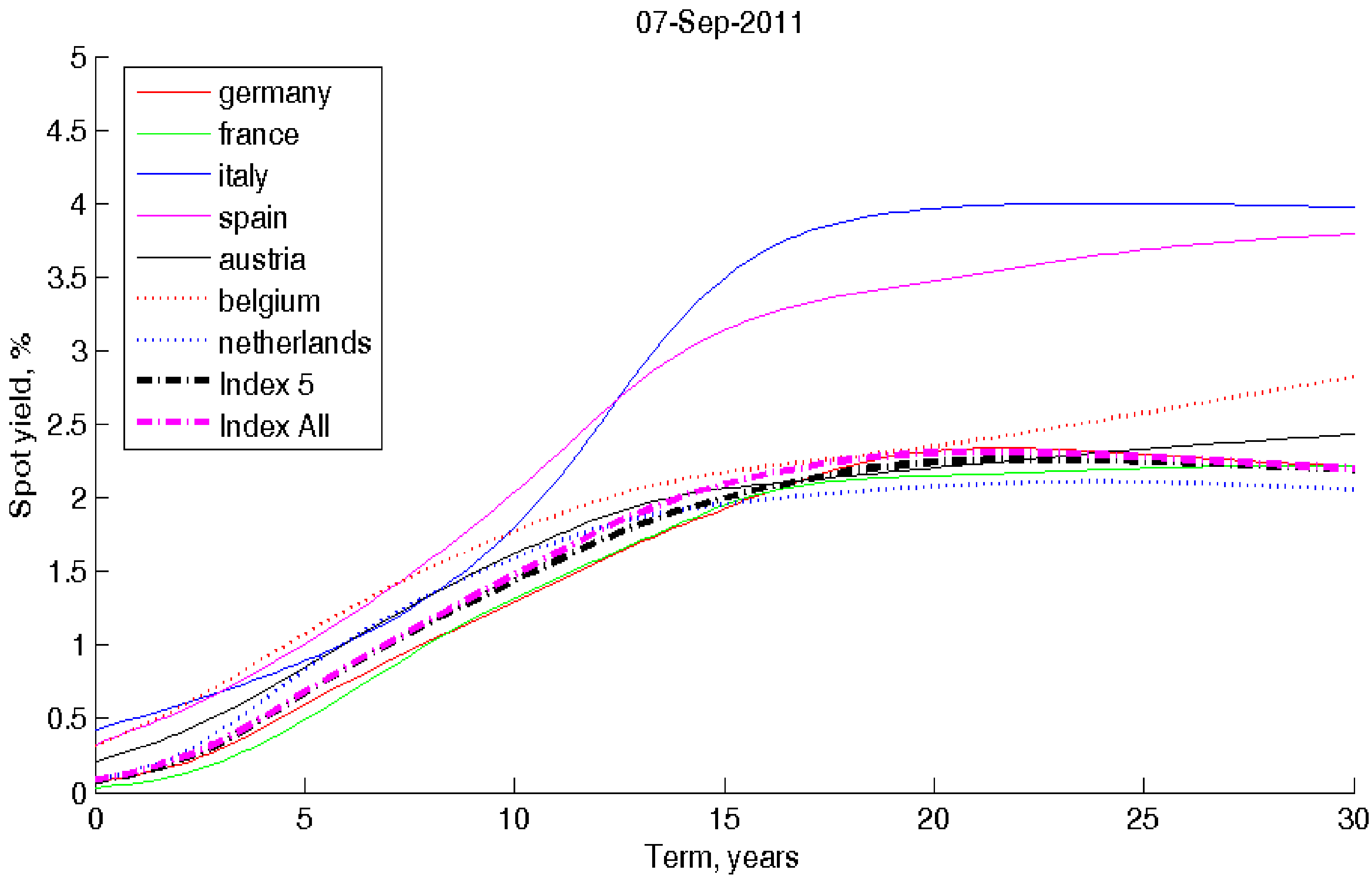
Dynamics of Country-Specific Estimates of Risk-Free Yield Curve (cont'd)

- At the middle of August 2011 the country-specific estimates approached again and this relation has been holding during August and September.

Dynamics of Country-Specific Estimates of Risk-Free Yield Curve (cont'd)



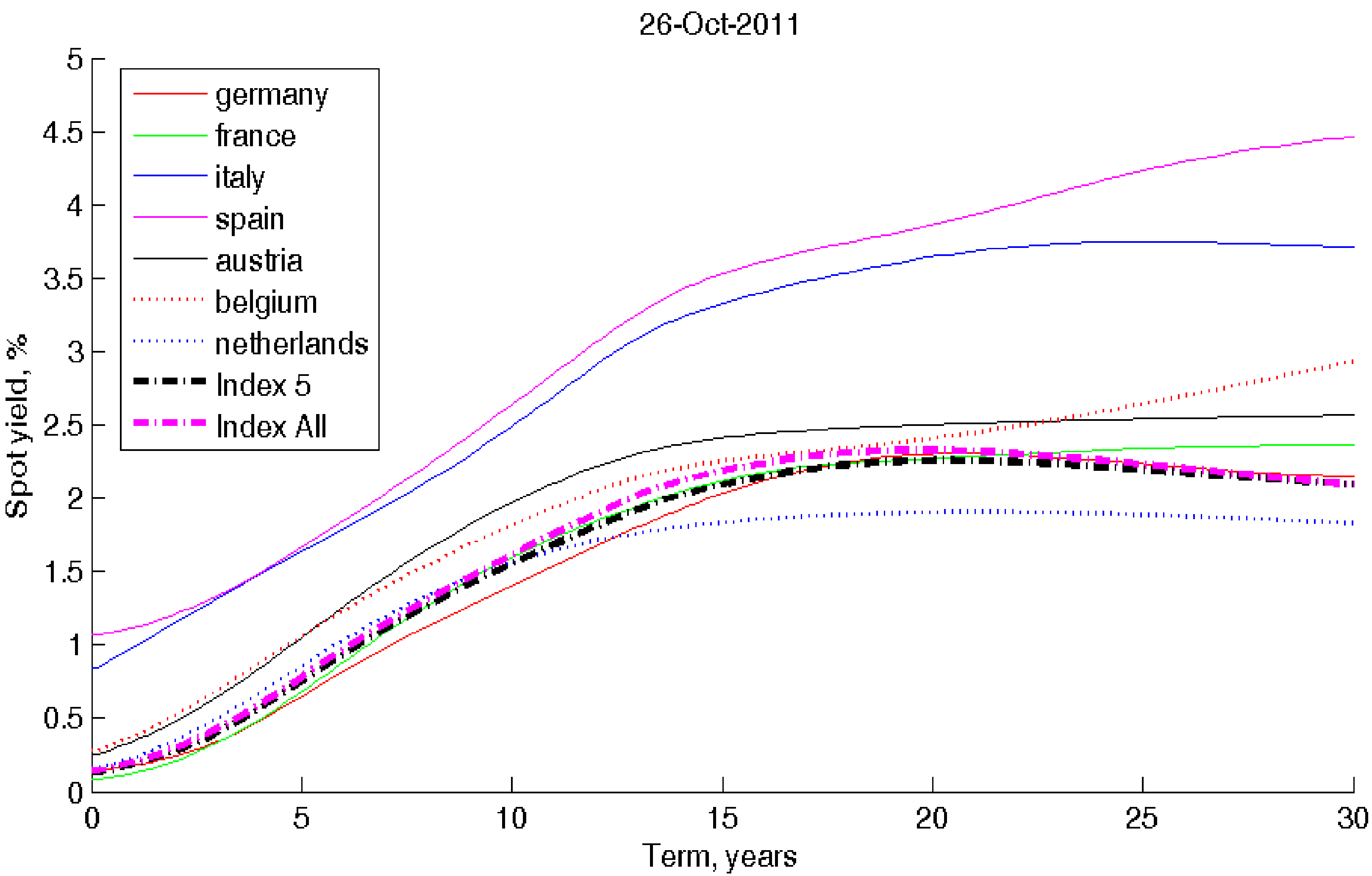
Dynamics of Country-Specific Estimates of Risk-Free Yield Curve (cont'd)



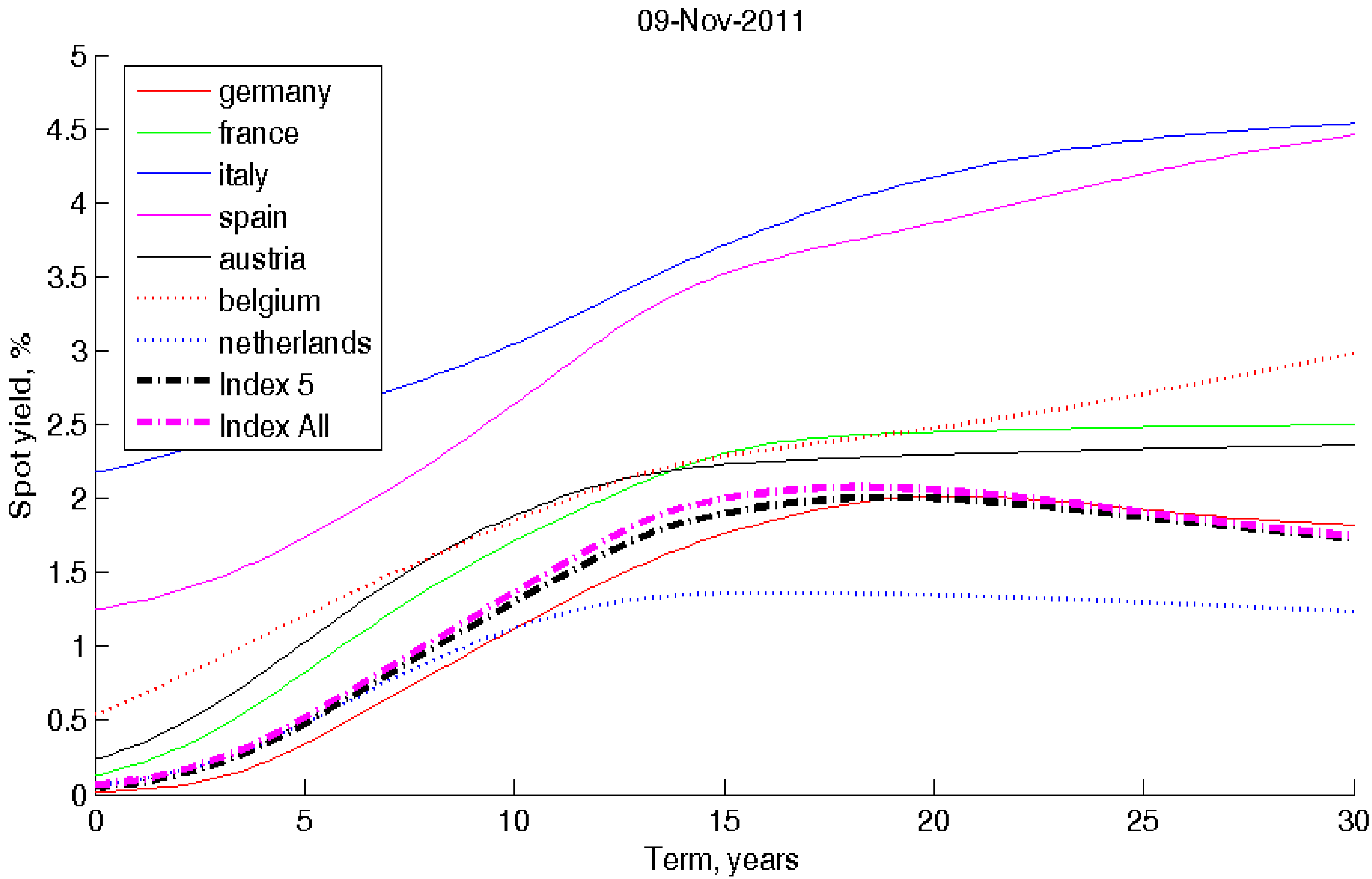
Dynamics of Country-Specific Estimates of Risk-Free Yield Curve (cont'd)

- However the divergence resumed in October 2011 and by the beginning of the November discrepancies between estimates became much lager then ever.

Dynamics of Country-Specific Estimates of Risk-Free Yield Curve (cont'd)



Dynamics of Country-Specific Estimates of Risk-Free Yield Curve (cont'd)



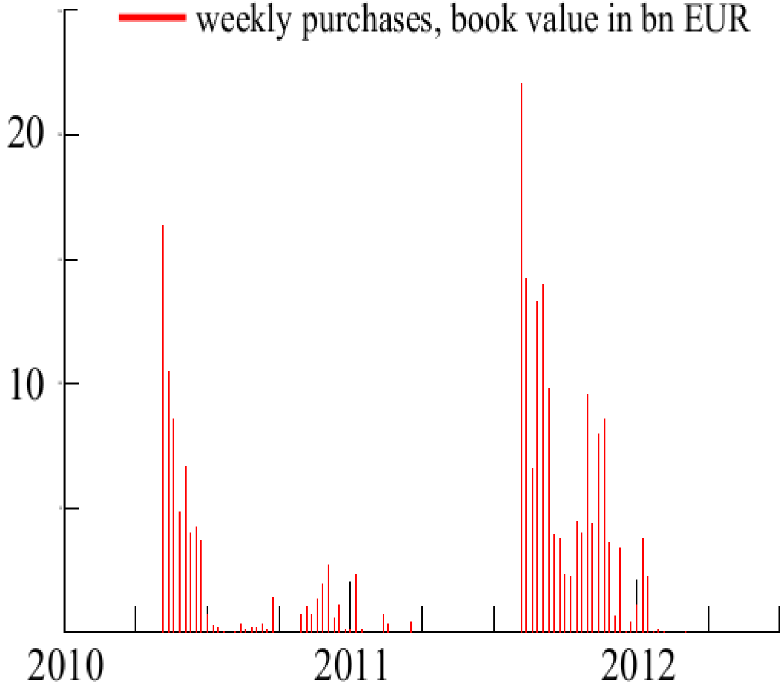
Securities Markets Programme

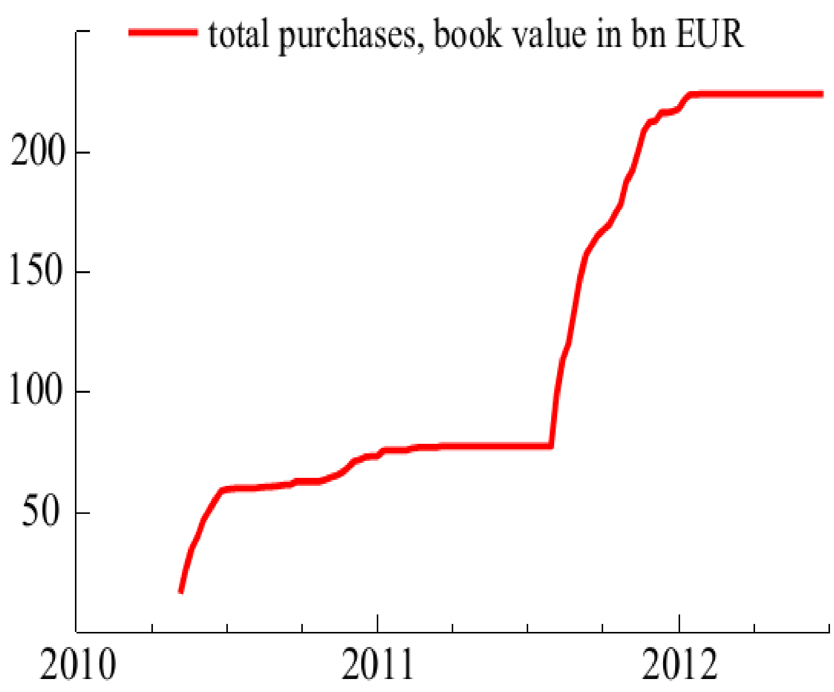
- Our data sample covers period of Securities Markets Programme (SMP) launched by ECB in May 2010 "to ensure depth and liquidity in those market segments which are dysfunctional. The objective of this programme [was] to address the malfunctioning of securities markets and restore an appropriate monetary policy transmission mechanism".

- As stated in working paper by ECB (2014), the Eurosystem started to intervene in the secondary markets of Greek, Irish and Portuguese government bond markets, and from August 2011 extended this to Italian and Spanish government bond markets. See working paper by ECB (2014).

- Observed dynamics of the country-specific estimate of risk-free curve can be explained by the ECB interventions in government bond markets.

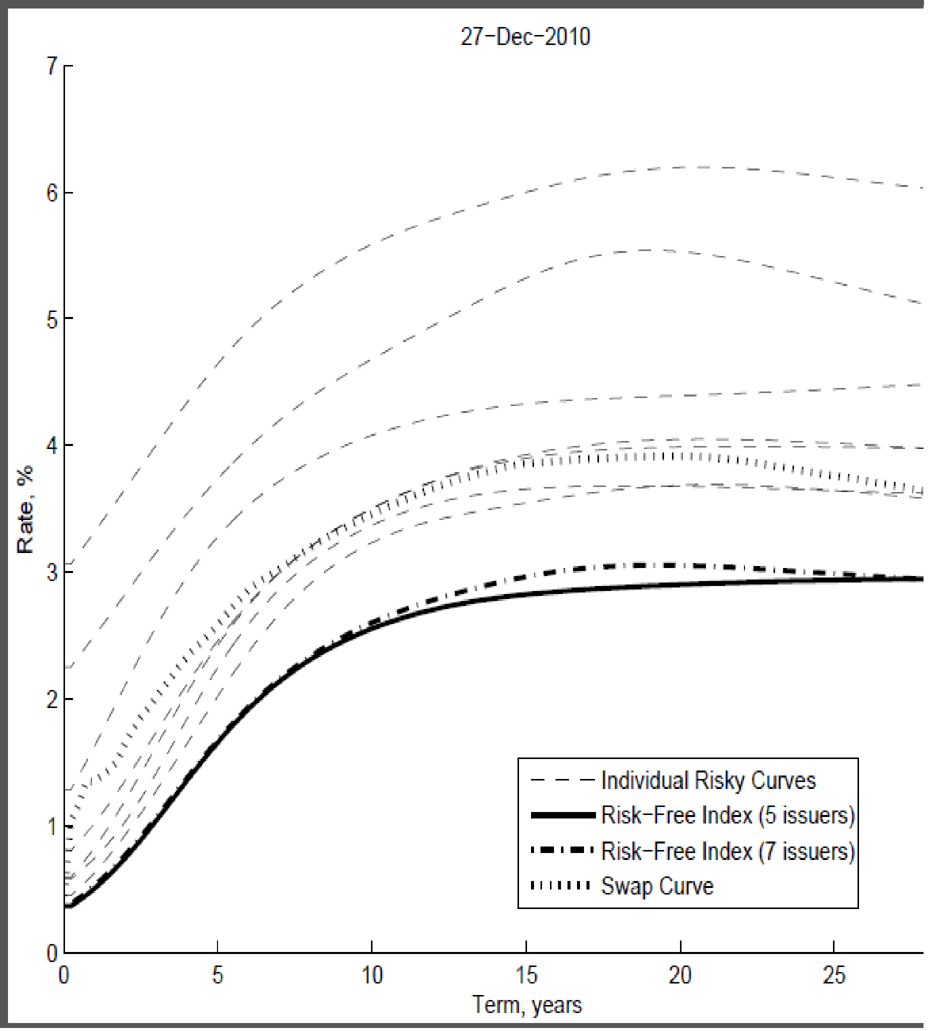
Securities Markets Programme (cont'd)

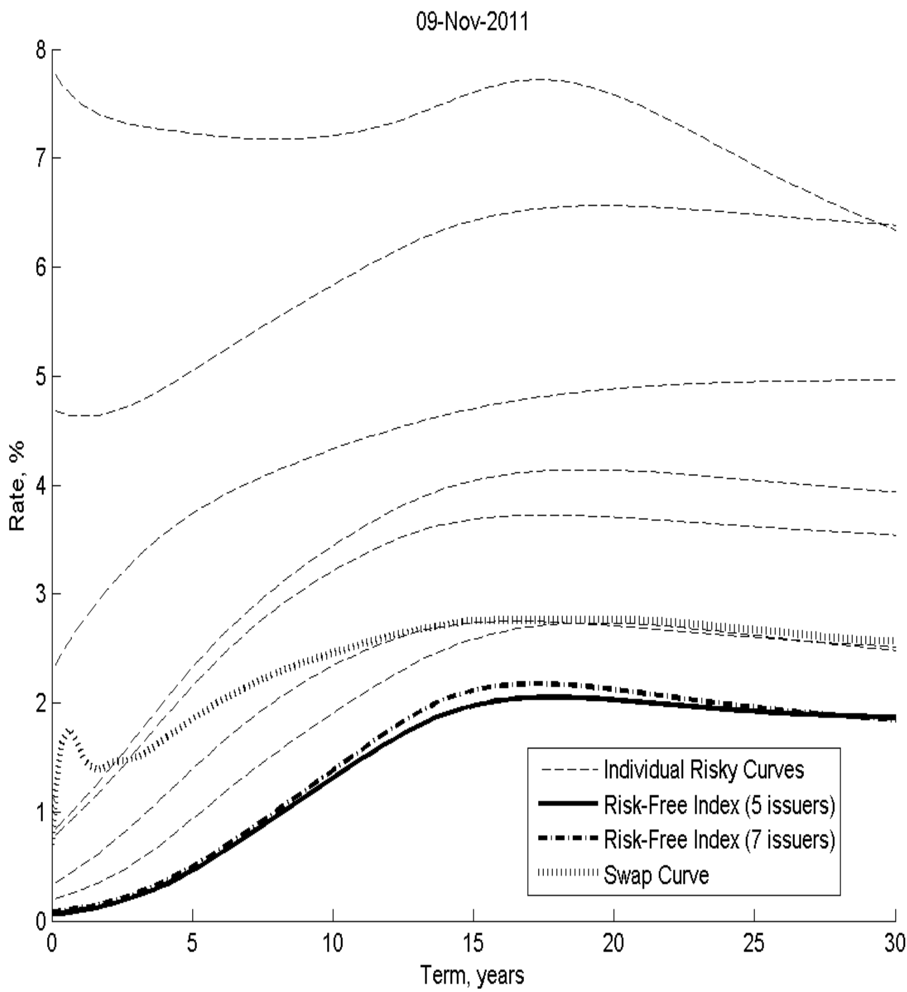




Source: Eser and Schwaab (2013) . Assessing asset purchases within the ECB ' s Securities Markets Programme. ECB Working Paper 1587, European Central Bank.

Consistency of Index Risk-Free Yield Curve





4. CONCLUSIONS

Conclusions

1. The straightforward approach do not allow obtaining the estimate of term structure of risk-free yield satisfying any reasonable degree of accuracy.

2. The rigorous procedure for selection of issuers and financial instruments has to be developed and applied to general set of sovereign bond and CDS data in order to ensure homogeneous base for index calculation.

Conclusions (cont'd)

3. The inherent accuracy of the construction of index risk-free curve has to be determined.

   There are few CDS contracts on any entity, the CDS maturities are evenly spread over time-line and their bid-ask quotes are fairly large in comparison with the respective bonds. Therefore, the CDS quotes are not really restrictive in construction of no-arbitrage bounds.

   In contrast, the bond quotes a very restrictive. In this case the bond issued by different countries should be limited to benchmarks and implied bid-ask spread should be determined for those benchmarks.

Thank you for your attention!

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