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Entrepreneurial Risks

Quantifying reflexivity in financial markets: towards a prediction of flash crashes

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Perm Winter School, Perm, Russia, February 2-4, 2012

The Story of The Efficient Market

Weak-form efficiency

- Future prices cannot be predicted by analyzing prices from the past (*no TA, but FA is possible*).

Semi-strong-form efficiency

- Share prices adjust to publicly available new information very rapidly and in an unbiased fashion, such that no excess returns can be earned by trading on that information (*no TA, no FA*).

Strong-form efficiency

- Share prices reflect all information, public and private, and no one can earn excess returns (*even insiders couldn't make it*).

One of conclusions of **Efficient Market Hypothesis (EMH)** is pure exogenous point of view on price formation. Price could change only under information revealed, crashes could be only exogenous.

- E. Fama (1970) "Efficient Capital Markets: a Review of Theory and Empirical Work." J. of Finance 25 (2): 383–417
- E. Fama (1991) "Efficient Capital Markets: II." Journal of Finance 46 (5): 1575–1617
- P. Samuelson (1973) "Proof That Properly Discounted Present Values of Assets Vibrate Randomly." The Bell Journal of Economics and Management Science 4 (2): 369–374
- P. Samuelson (2006) "Proof That Properly Anticipated Prices Fluctuate Randomly." Ind. Manag. Review 6: 41–49

Rational agents

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In its original form the Efficient Market Story assumes presence of the **Rational Agents (Investors)**, who

- are perfectly informed about the politico-economical situation in the world
- can immediately process all new information and reveal arbitrage opportunities
- can profit even from tiny mispricing, moving price to its "fundamental" value

Rational Agents

- are always concentrated
- never become tired
- never feel greedy or scary
- never make mistakes or even sub-optimal decisions..

“Rational agents”

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Rational agents vs rational expectations

- The modern concept of EMH doesn't require all agents to be rational. It was shown that prices do fully reflect all available information if investors have "**rational expectations**" (Lucas, 1978). E.g. individually they could be irrational but, behave rational *in average*, as if all agents were rational.

- Nevertheless this "collective rationality" also poses many problems when compared with reality. Even being rational individually, we behave irrationally as a crowd facing such collective effects as **imitation**, **informational cascades** etc. that result in **herding**.

• R. Lucas (1978) "Asset prices in an exchange economy" Econometrica 46 (6): 1429–1445
See also:
• D. Sornette (2002) "Why Stock Markets Crash: Critical Events in Complex Financial Systems" Princeton Univ. Press

Imitation

Imitation (observation and replication of someone's behavior) is among the most complex forms of learning.
It is found in highly socially living species which show, from a human observer point of view, "intelligent" behavior and signs for the evolution of traditions and culture.

Informational cascades

Being in the crowd infer information and limits rationality. People observe actions of others and then make the same choice that the others have made, independently of their own private information signals.

"Well, heck! If all you smart cookies agree, who am I to dissent?"

- S. Bikhchandani, D. Hirshleifer, I. Welch (2008) "Information cascades" The New Palgrave Dictionary of Economics, 2nd ed.

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Thy Neighbor's Portfolio: Word-of-Mouth Effects in the Holdings and Trades of Money Managers

HARRISON HONG, JEFFREY D. KUBIK, and JEREMY C. STEIN*

ABSTRACT

A mutual fund manager is more likely to buy (or sell) a particular stock in any quarter if other managers in the same city are buying (or selling) that same stock. This pattern shows up even when the fund manager and the stock in question are located far apart, so it is distinct from anything having to do with local preference. The evidence can be interpreted in terms of an epidemic model in which investors spread information about stocks to one another by word of mouth.

In this paper, we explore the hypothesis that investors spread information and ideas about stocks to one another directly, through word-of-mouth communication. This hypothesis comes up frequently in informal accounts of the behavior of the stock market.[1] For example, in his bestseller *Irrational Exuberance*, Shiller (2000) devotes an entire chapter to the subject of "Herd Behavior and Epidemics," and writes

> A fundamental observation about human society is that people who communicate regularly with one another think similarly. There is at any place and in any time a *Zeitgeist*, a spirit of the times. . . . Word-of-mouth transmission of ideas appears to be an important contributor to day-to-day or hour-to-hour stock market fluctuations. (pp. 148, 155)

Have we seen any real-life examples?

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Financial bubbles, which we have been observing for over 400 years:

Tulip mania

South Sea bubble

IT bubble

Housing bubble

How efficient is the **real** market?

“Excess volatility” puzzle

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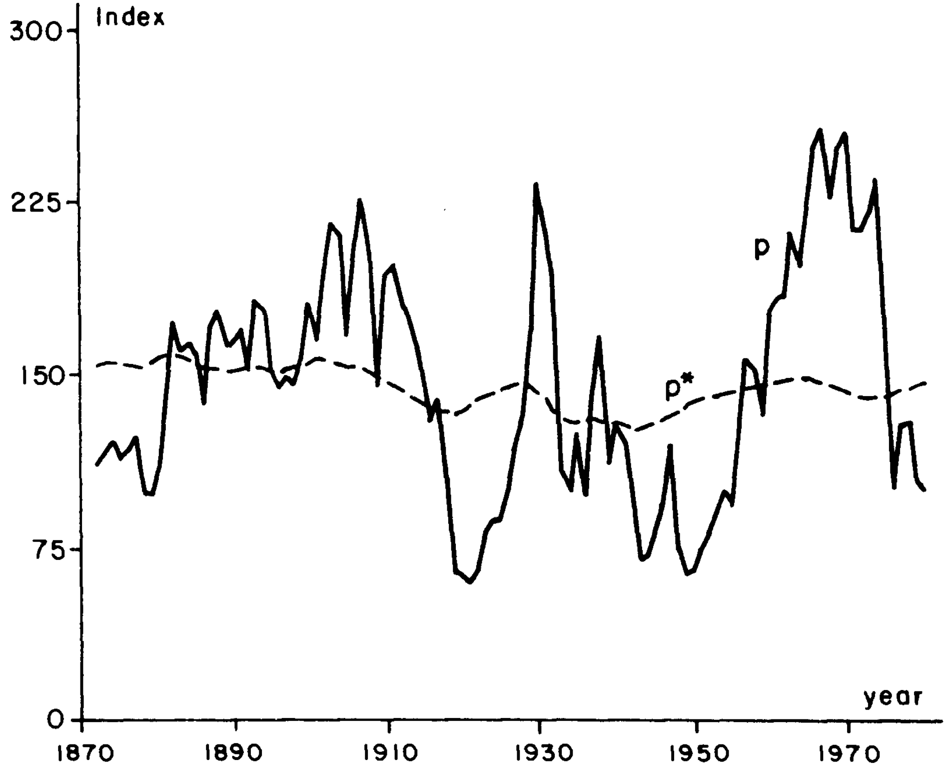
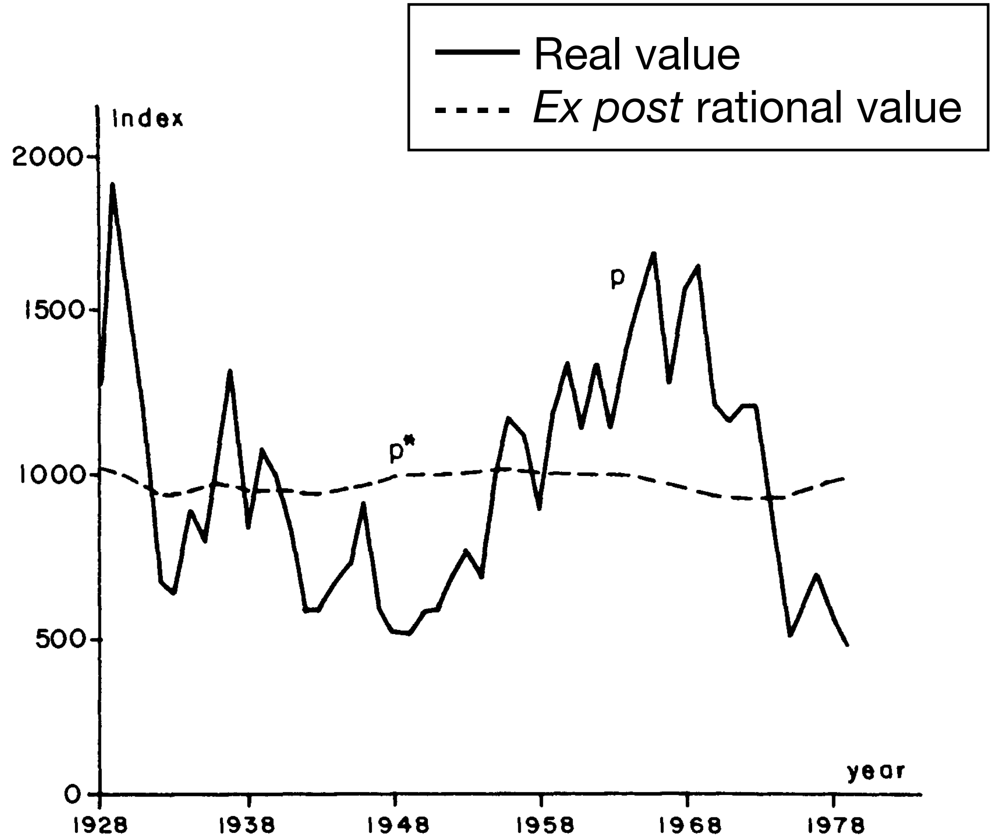


S&P 500



Dow Jones Industrial Average

R. Shiller (1981) “Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?” The American Economic Review 71 (3): 421–436

See also:

- S. F. LeRoy and R.D. Porter (1981) “The Present-Value Relation: Tests Based on Implied Variance Bounds.” Econometrica: Journal of the Econometric Society: 555–574
- S. F. LeRoy (2008) “Excess Volatility Tests.” The New Palgrave Dictionary of Economics, 2nd ed.

"What moves stock prices?"

Major events and changes in S&P500 Index, 1941-1987

Event	Date	Percent Change
Japanese bomb Pearl Harbor	Dec. 8, 1941	−4.37
US declares war against Japan	Dec. 9, 1941	−3.23
Roosevelt defeats Dewey	Nov. 8, 1944	−0.15
Roosevelt dies	Apr. 13, 1945	1.07
Atomic bombs dropped on Japan:		
Hiroshima bomb	Aug. 6, 1945	0.27
Nagasaki bomb; Russia declares war	Aug. 9, 1945	1.65
Japanese surrender	Aug. 17, 1945	−0.54
Truman defeats Dewey	Nov. 3, 1948	−4.61
North Korea invades South Korea	June 26, 1950	−5.38
Truman to send US troops	June 27, 1950	−1.10
Eisenhower defeats Stevenson	Nov. 5, 1952	0.28
Eisenhower suffers heart attack	Sep. 26, 1955	−6.62
Eisenhower defeats Stevenson	Nov. 7, 1956	−1.03
U-2 shot down; US admits spying	May 9, 1960	0.09
Kennedy defeats Nixon	Nov. 9, 1960	0.44
Bay of Pigs invasion announced;	Apr. 17, 1961	0.47
Details released over several days	Apr. 18, 1961	−0.72
	Apr. 19, 1961	−0.59

Largest post-war movements in S&P 500 Index and their "causes"

	Date	Percent Change	New York Times Explanation*
1	Oct. 19, 1987	−20.47	Worry over dollar decline and trade deficit, fear of US not supporting dollar.
2	Oct. 21, 1987	9.10	Interest rates continue to fall; deficit talks in Washington; bargain hunting.
3	Oct. 26, 1987	−8.28	Fear of budget deficits; margin calls; reaction to falling foreign stocks.
4	Sep. 3, 1946	−6.73	"No basic reason for the assault on prices."
5	May 28, 1962	−6.68	Kennedy forces rollback of steel price hike.
6	Sep. 26, 1955	−6.62	Eisenhower suffers heart attack.
7	Jun. 26, 1950	−5.38	Outbreak of Korean War.
8	Oct. 20, 1987	5.33	Investors looking for "quality stocks."
9	Sep. 9, 1946	−5.24	Labor unrest in maritime and trucking industries.
10	Oct. 16, 1987	−5.16	Fear of trade deficit; fear of higher interest rates; tension with Iran.
11	May 27, 1970	5.02	Rumors of change in economic policy. "The stock surge happened for no fundamental reason."

D. Cutler, J. Poterba, L. Summers (1987) "What moves stock prices?" Journal of Portfolio Management 15 (3): 4–12

See also:

- G. McQueen, V. Roley (1993) "Stock prices, news, and business conditions." Review of Fin. Studies 6 (3): 683–707
- O. Erdogan, A. Yezegel (2009) "The news of no news in stock markets", Quantitative Finance 9 (8): 897–909
- M. Fleming, E. Remolona (1997) "What Moves the Bond Market?" FRBNY Economic Policy Review 3 (4): 31–50
- R. C. Fair (2002) "Events That Shook the Market." Journal of Business 75 (4): 713–731

Exogenous versus endogenous jumps in HF

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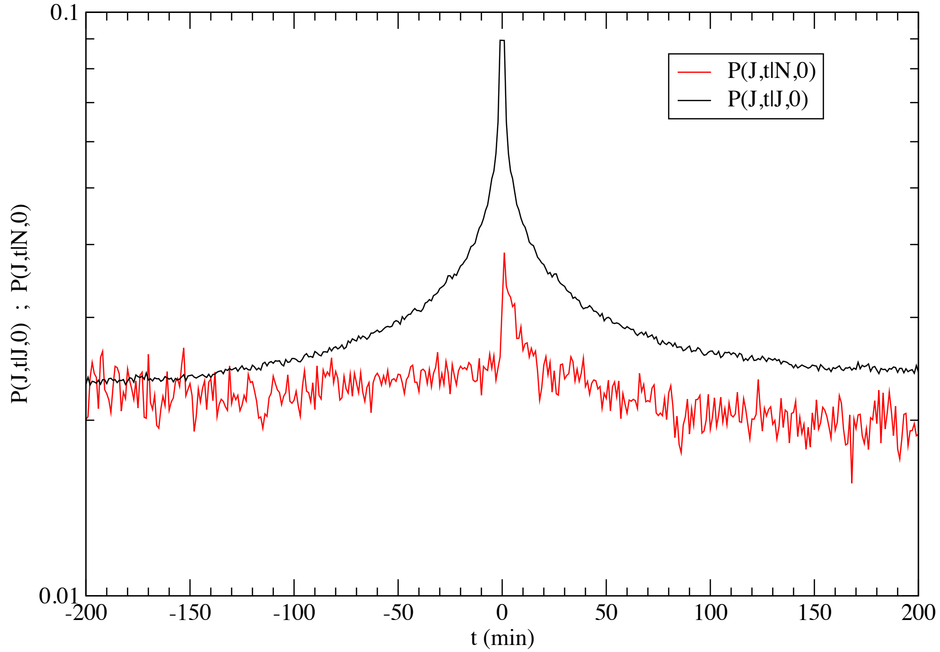
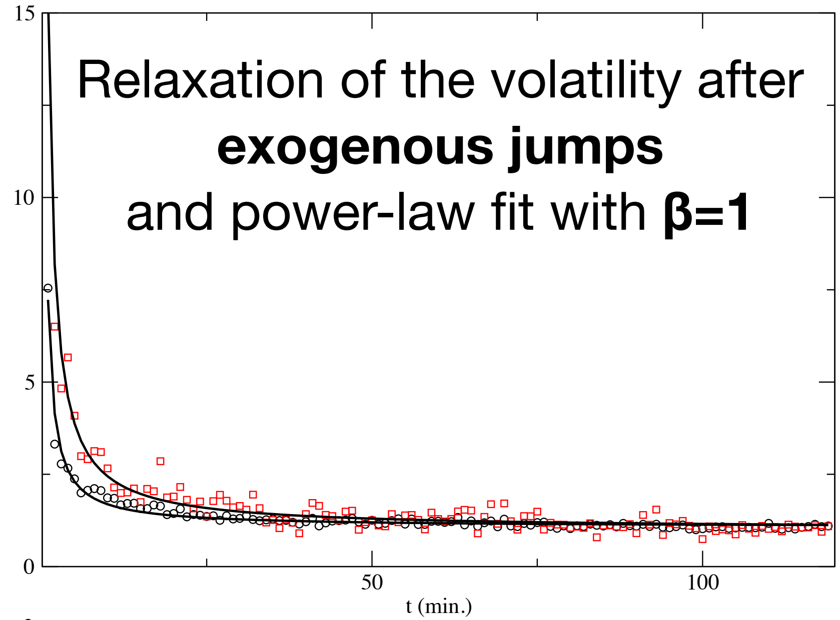
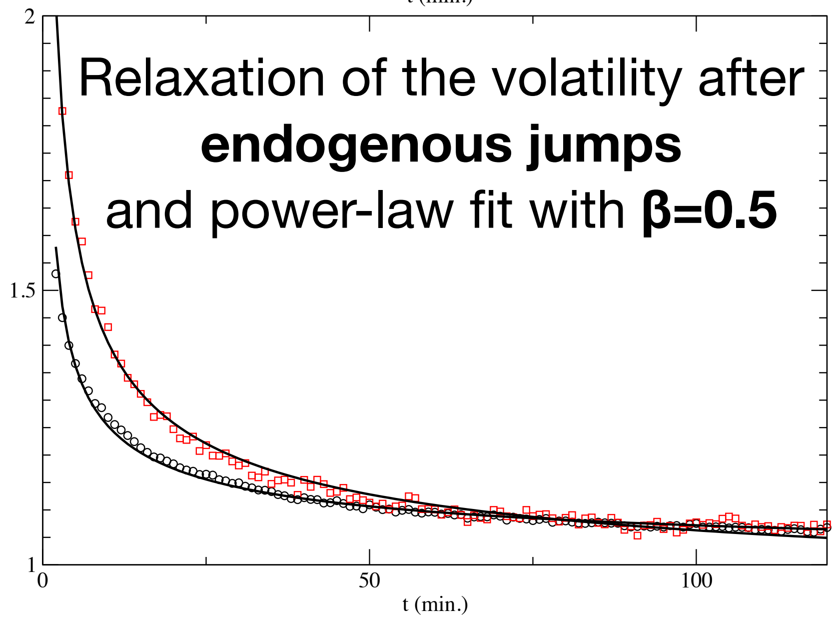


Probability of having **jump** in price at time *t*, conditional on:
(red) **news** at time *t*=0
(black) **jump** at time *t*=0

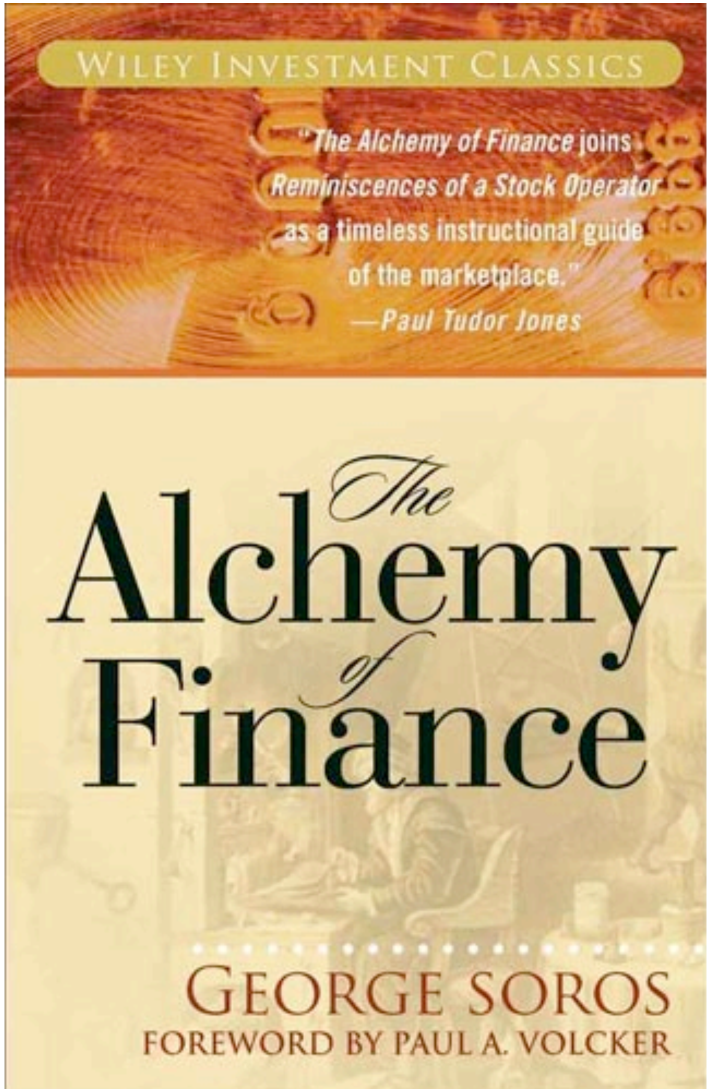




A. Joulin, A. Lefevre, D. Grunberg, J.-P. Bouchaud (2008)
"Stock price jumps: news and volume play a minor role." Wilmott Magazine, Sep/Oct: 46.

Idea of the “reflexivity” of market

- George Soros adapted ideas of Orlean (1980x) and others and proposed the concept of “**reflexivity**”, where the biases of individuals enter into market transactions, potentially changing the perception of fundamentals of the economy.
- When markets are rising or falling rapidly, they are typically in the state of *disequilibrium* rather than *equilibrium*, and that the conventional economic theory of the market (EMH) is not valid in these situations.
- Reflexivity asserts that *prices do in fact influence the fundamentals* and that these newly-influenced set of fundamentals then proceed to change expectations, thus influencing prices; the process continues in a self-reinforcing pattern.
- In other words, the underlying market mechanisms create **positive feedback loops** that cause prices to diverge from equilibrium.



Target

We use a class of **self-excited models** that combines (i) external influence on system with (ii) feedback mechanisms to test in a technical way the hypothesis of reflexivity (endogeneity) of the market.

We will show that market is operating close to **criticality**, implying **significant role of endogenous feedback** mechanisms in price formation process and increase of this role over last 10 years due to growth of AT and HFT.

Moreover we will introduce the **metric** that allow one to estimate quantitatively the **relative proportion** between endogenous and exogenous price movements.

We will illustrate the power of this **metric** in distinguishing between **exogenous** (triggered by news) and **endogenous** (resulted from feedback) shocks on market.

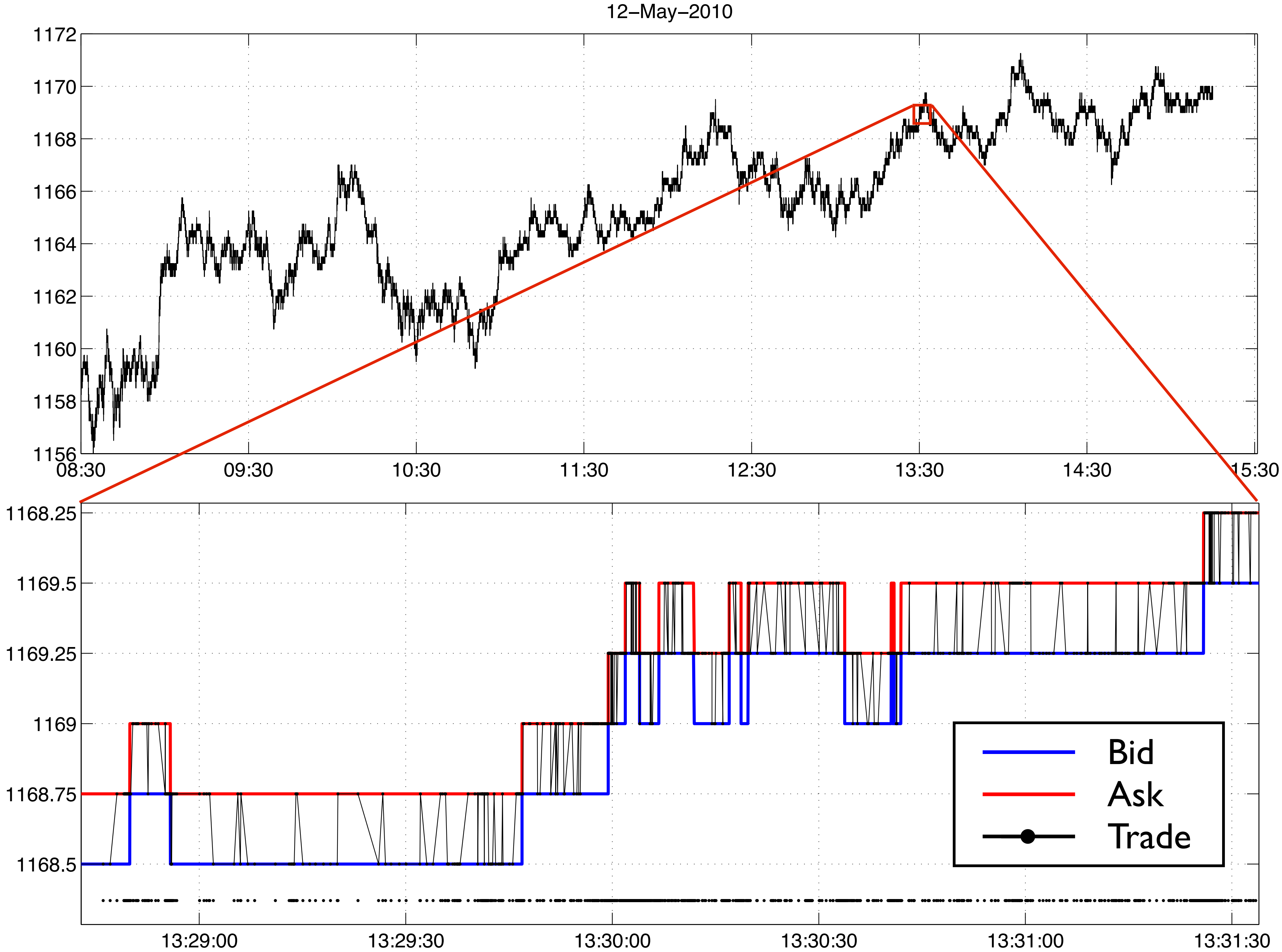
Data: E-mini S&P 500 Futures

E-mini S&P 500 Futures Contract represents a fraction of the normal S&P 500 futures contract.

Some facts:

- Ticker symbol: ES
- Exchange: Chicago Mercantile Exchange
- Contract months: Five months in the March Quarterly Cycle (Mar, Jun, Sep, Dec)
- Trading time: 23.25 hours/day (active trading: 6.75 hours)
- Contract size: $50 x E-mini S&P 500 futures price
- Tick size: 0.25 index points=$12.50
- Initial margin: $5,625
- **Average daily volume in 2010: 2,194,975** (for comparison: average daily volume of regular S&P 500 futures: 345,483)

Sample series of E-mini's



Order-book dynamics

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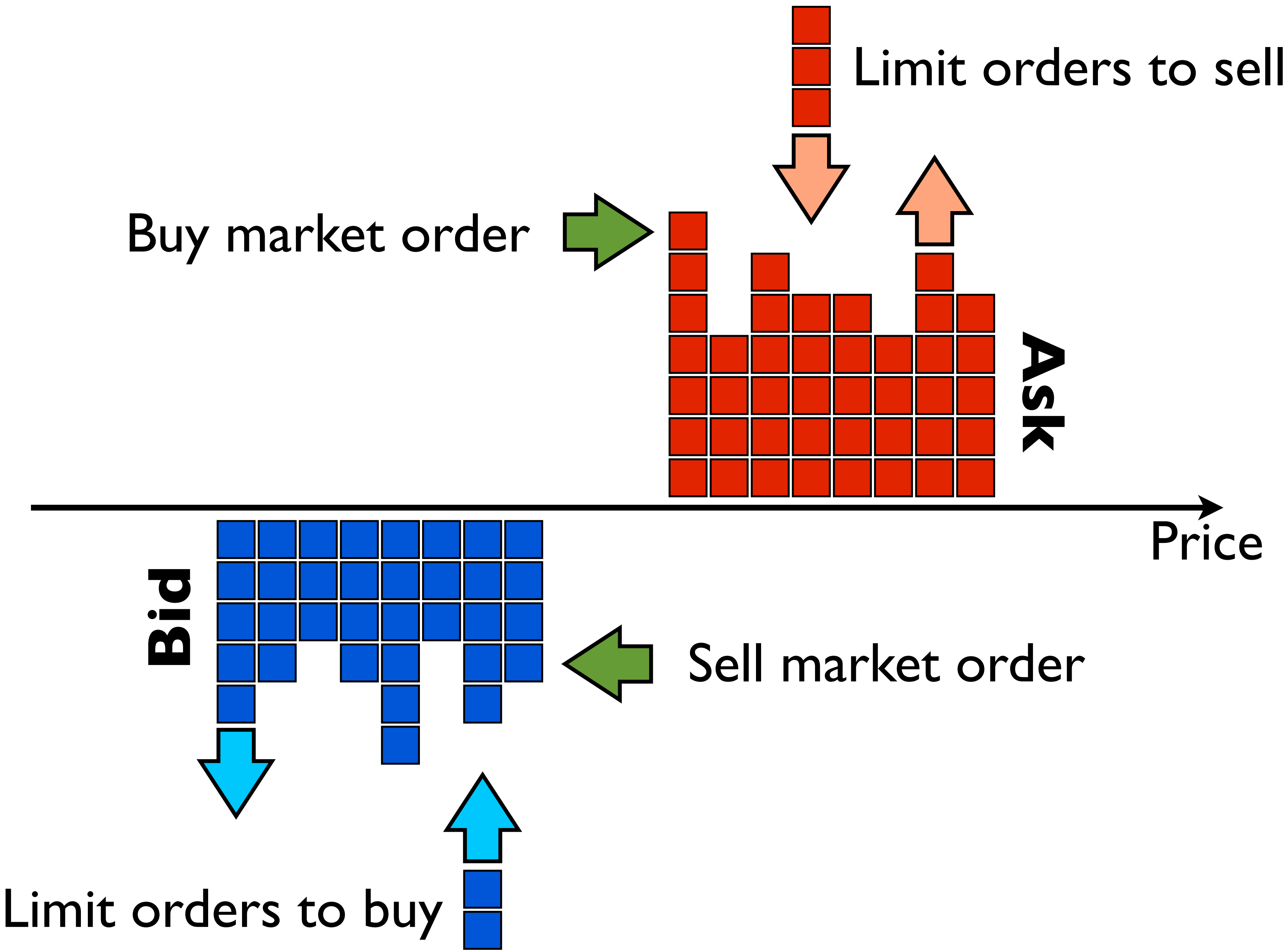


Null hypothesis: **Random Walk** (Bachelier, 1900).

Stylized facts of real price time series:

- *Absence of returns' autocorrelations*
- *Aggregational Gaussianity*
- Fat tails of distributions
- Long memory in volatility
- Intermittency and Volatility clustering
- Multifractal scaling
- Time reversal asymmetry and Leverage effect
- Gain/Loss asymmetry
- Asymmetry in time scales
- Volume-Volatility correlation
- Bubbles and crashes

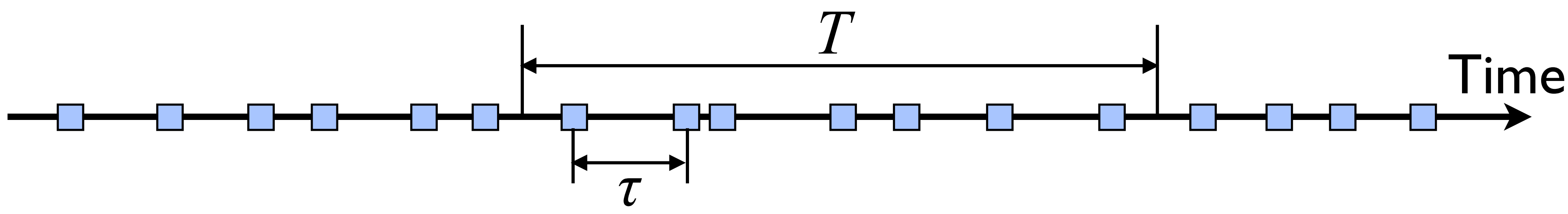
Reject null

Null hypothesis: **Poisson process.**

Poisson process is a point process for which number of events in a given time interval T is independent from events outside the interval and described with Poisson distribution:

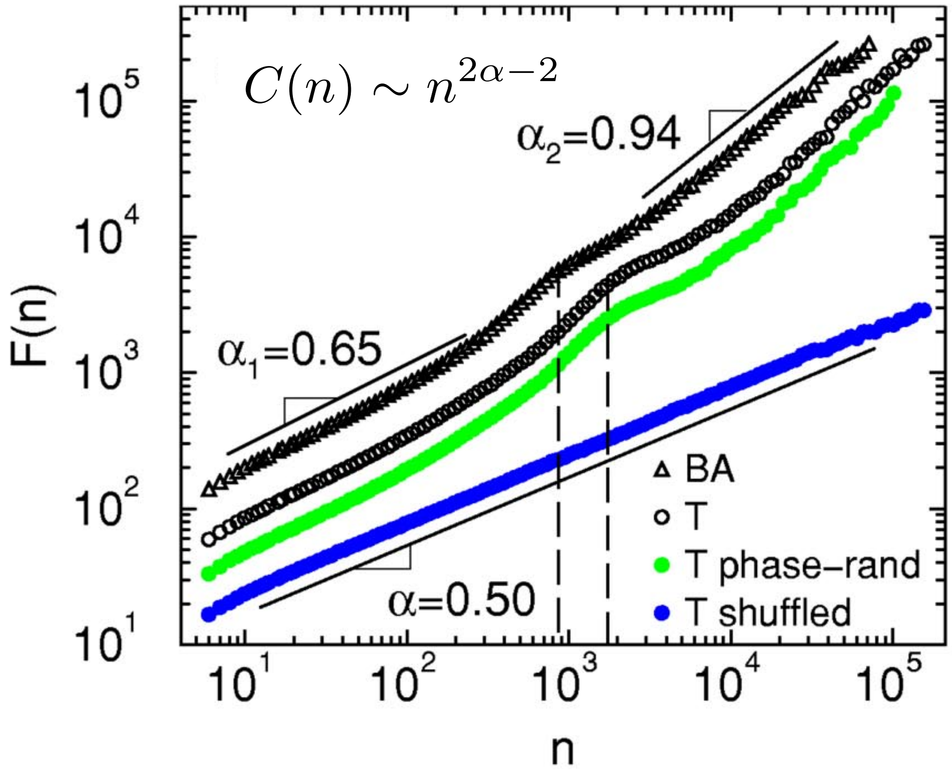
$$P\Big[N(t+T) - N(t) = k\Big] = \frac{(\lambda T)^k}{k!} e^{-\lambda T}$$

where λ=const is an *intensity* of the process.

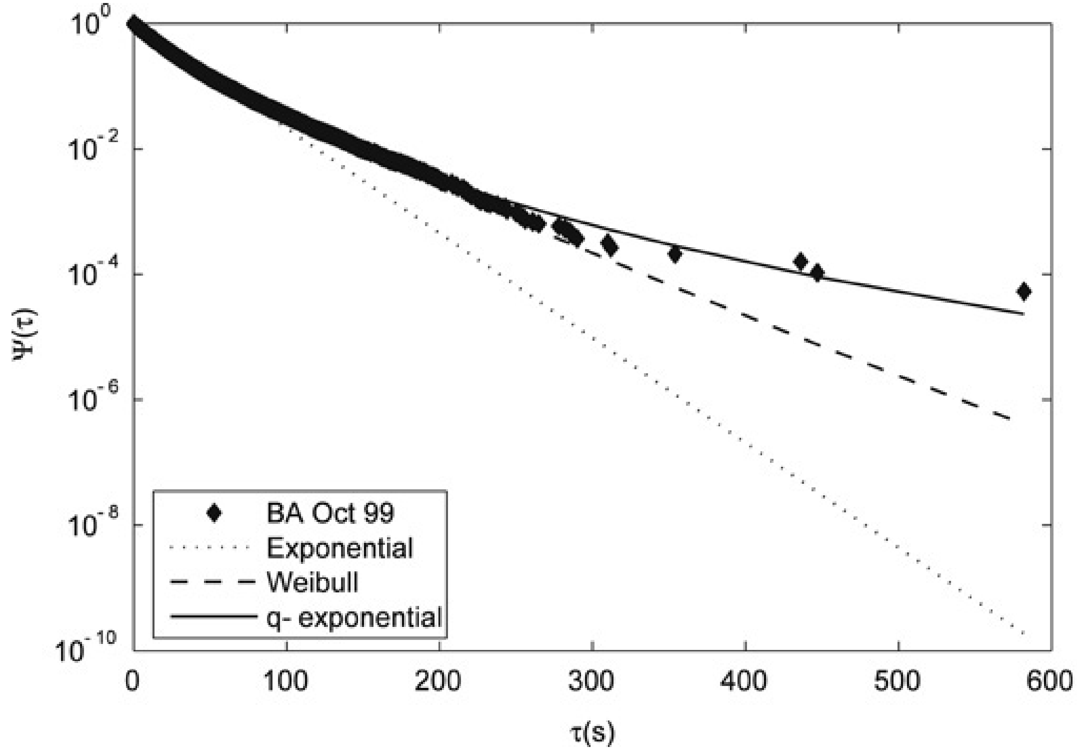


Poisson process is characterized by an exponential distribution of inter-event times τ.

"Stylized facts" of real order arrivals



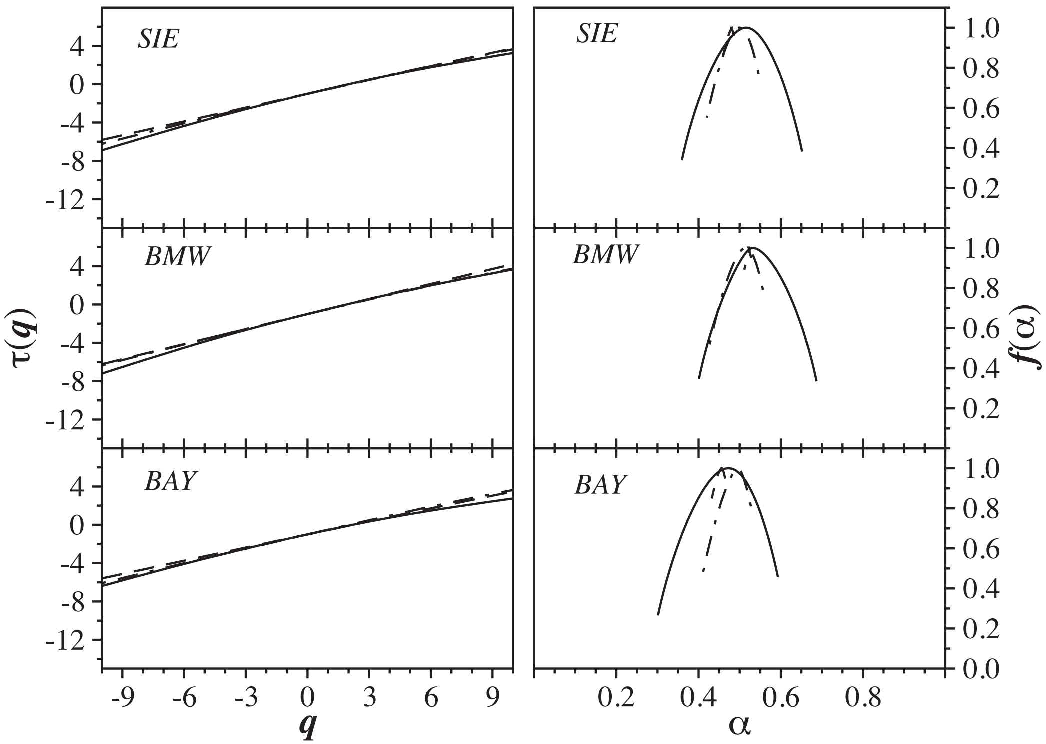
Clustering of order arrivals
Long memory in inter-trade intervals



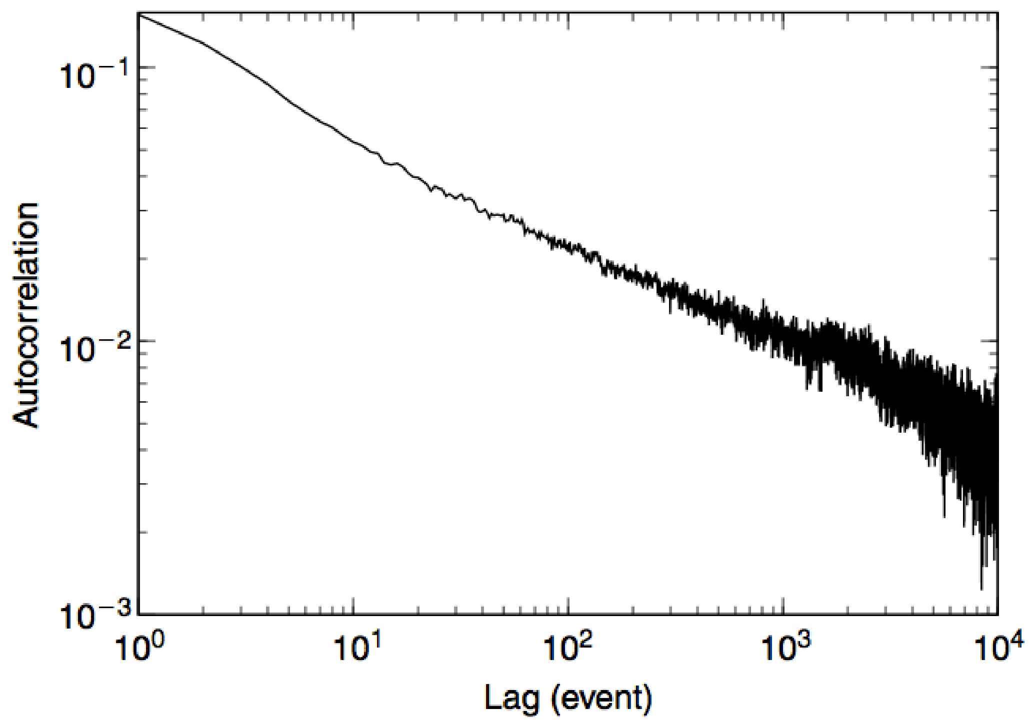
Slower-than-exponential decay of the distribution of inter-trade intervals

- P. Ivanov, A. Yuen, B. Podobnik, Y. Lee (2004) "Common Scaling Patterns in Intertrade Times of U. S. Stocks." Physical Review E 69 (5): 056107.
- M. Politi, E. Scalas (2008) "Fitting the Empirical Distribution of Intertrade Durations." Physica A, 387 (8-9): 2025–2034.

“Stylized facts” of real order arrivals



Multifractal scaling of inter-trade intervals



Long memory in the signs of orders

- P. Oswiecimka, J. Kwapien, S. Drozdz (2005) “Multifractality in the Stock Market: Price Increments Versus Waiting Times.” Physica A, 347: 626–638.
- J.-P. Bouchaud, D. Farmer, F. Lillo (2008) “How Markets Slowly Digest Changes in Supply and Demand.” In Handbook of Financial Markets: Dynamics and Evolution (Handbooks in Finance), 57–160.

Extended models:

- **Clustered point processes**
  Poisson process supplemented with artificial clusters around immigrants.

- **Autoregressive Conditional Durations (ACD)**
  GARCH-type model for the inter-trade intervals:

$$\tau_k = \theta_k z_k, \quad z_k > 0, \quad \mathrm{E}[z_k] = 1$$

$$\theta_k = \alpha_0 + \sum_{i=1}^{q} \alpha_i \tau_{k-i} + \sum_{i=1}^{p} \beta_i \theta_{k-i}$$

- **Self-excited point processes**:
  - Linear: **Hawkes process**
  - Nonlinear, e.g. **Multifractal Stress Activation (MSA)**

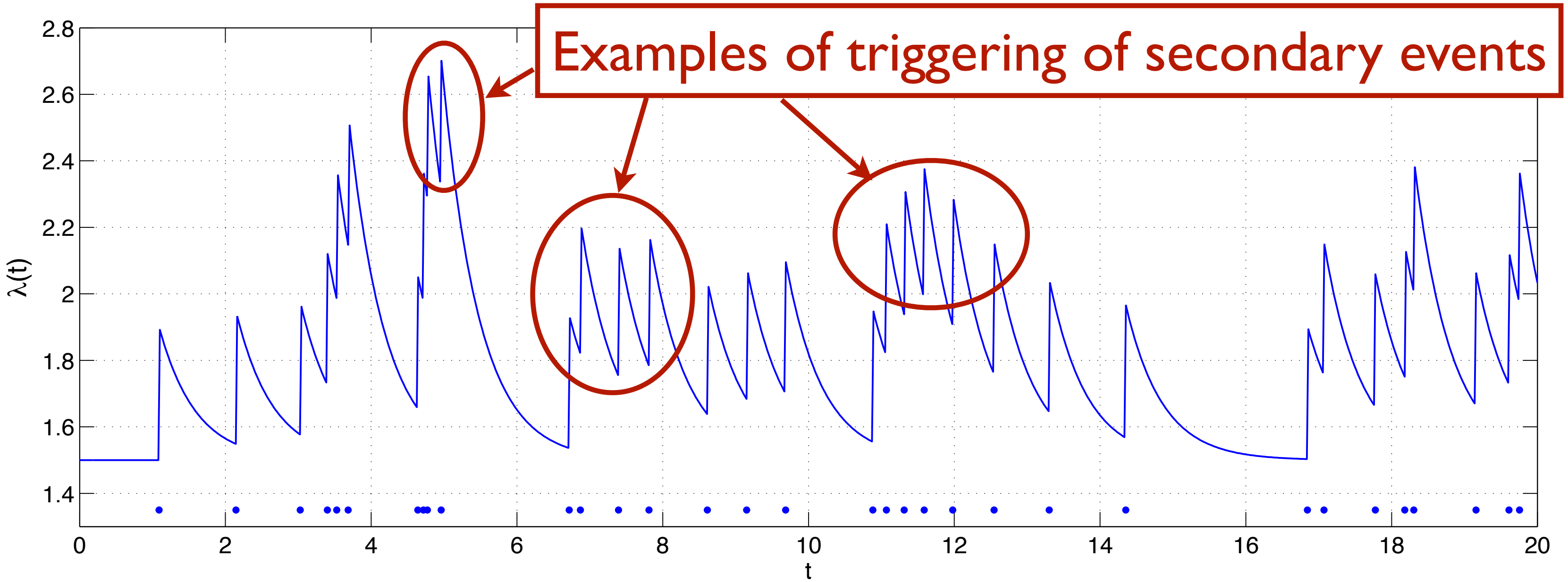
- R. Engle, J. Russell (1998) "Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data." Econometrica: Journal of the Econometric Society, 66 (5): 1127–1162.
- A. Hawkes (1971) "Point Spectra of Some Mutually Exciting Point Processes." Journal of the Royal Statistical Society. Series B (Methodological) 33 (3): 438–443.
- D. Sornette, G. Ouillon (2005) "Multifractal Scaling of Thermally Activated Rupture Processes." PRL 94(3): 038501

Self-excited Hawkes process

Self-excited Hawkes process is the point process whose intensity $\lambda_t(t)$ is conditional on its history:

$$\underbrace{\lambda(t) = \mu}_{\text{Background intensity}} + \sum_{t_i<t} \underbrace{\varphi(t-t_i)}_{\text{Self-excitation part}}$$

Traditionally the exponential memory kernel is used: $\varphi(t) = \alpha e^{-\beta t}$



Sample realization of the Hawkes process with μ=1.5, α=0.4, β=2

Branching representation of the Hawkes process

Recall that sum of N independent Poisson processes with intensities $\lambda_1, \lambda_2, ..., \lambda_N$ is a Poisson process with intensity $\lambda = \lambda_1 + \lambda_2 + ... + \lambda_N$.

Thus self-excited Hawkes process

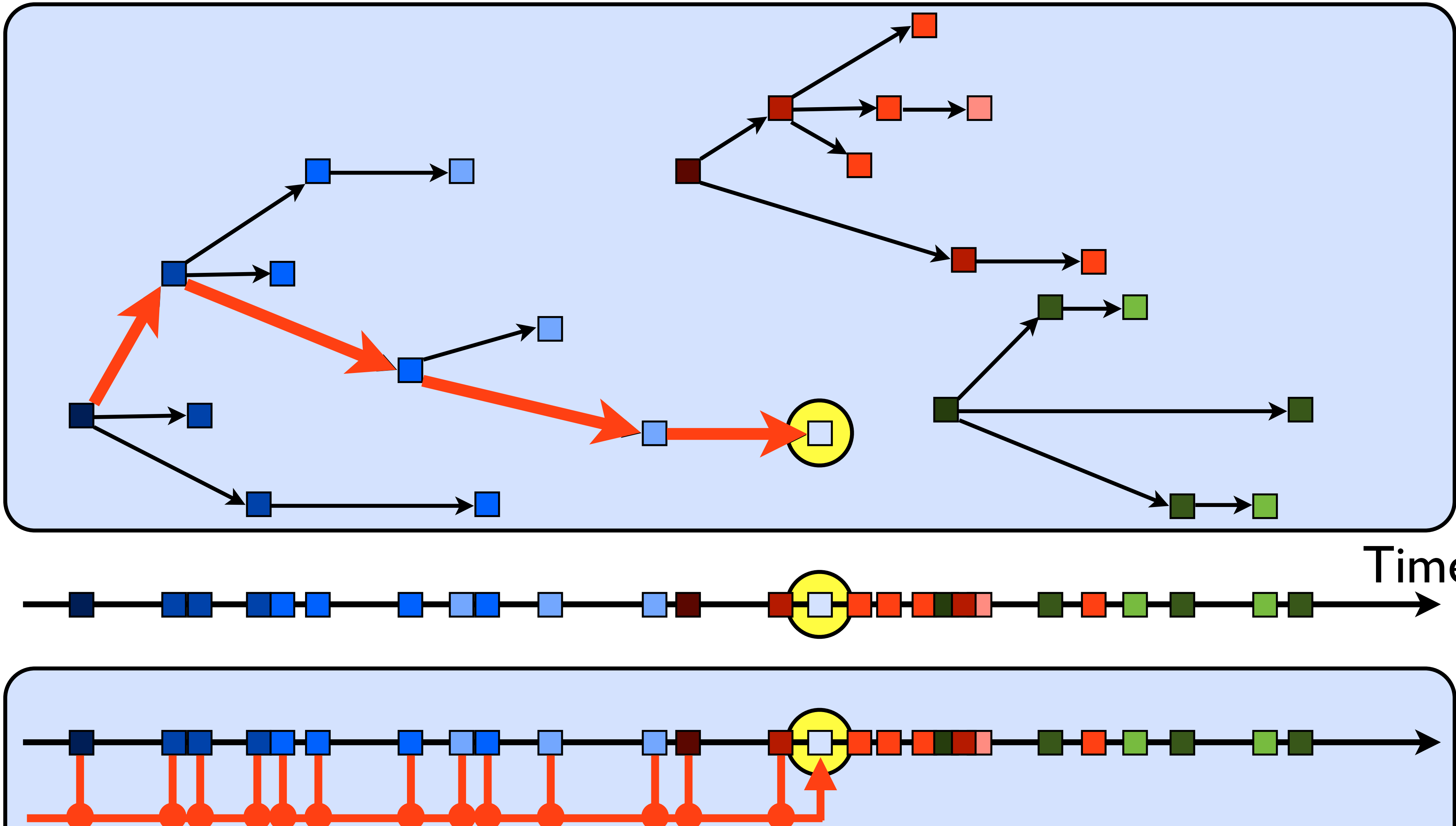
$$\lambda(t) = \mu + \sum_{t_i < t} \varphi(t - t_i)$$

could be regarded as a sum of independent non-homogeneous Poisson processes with intensities:

$$\lambda_0 = \mu, \quad \lambda_1 = \varphi(t - t_1), \quad \ldots, \quad \lambda_n = \varphi(t - t_n)$$

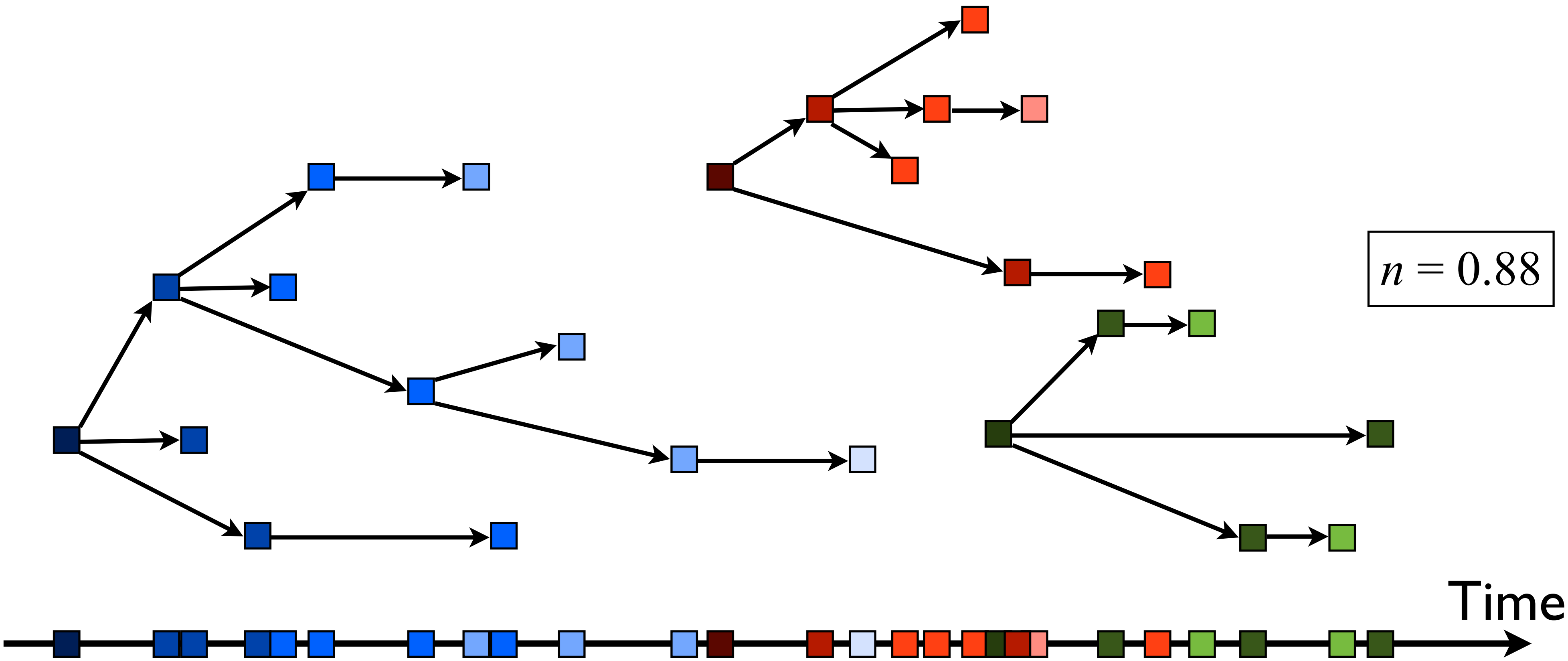
Each of them represent decaying intensity after a single shock and they altogether form a **branching process**.

Branching processes



$$\lambda(t) = \mu + \sum_{t_i < t} \varphi(t - t_i)$$

Branching processes



Crucial parameter of the branching process is the **"branching ratio" (n)** - average number of "daughters" per one "mother"

For $n < 1$ system is **subcritical** (stationary evolution)

For $n = 1$ system is **critical** (tipping point)

For $n > 1$ system is **supercritical** (with $p>0$ will explode to infinity)

The branching ratio

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In the *sub-critical regime* ($n < 1$), in the case of a constant background intensity ($\mu(t)=\mu=\text{const}$) the rate of all endogenous events ("aftershocks") is equal to:

$$R_{\text{endo}} = \mu \cdot \left(n + n^2 + n^3 + \ldots\right) = \mu \sum_{i=1}^{\infty} n^i = \frac{\mu n}{1-n}$$

The rate of exogenous events is equal to $R_{\text{exo}}=\mu$.

Total rate of all events is:

$$R = R_{\text{exo}} + R_{\text{endo}} = \mu + \frac{\mu n}{1-n} = \frac{\mu}{1-n}$$

Thus:

$$\boxed{n = R_{\text{endo}}/R}$$

In other words, **the branching ratio (n) is equal to the proportion of the average number of *endogenously* generated events among all events.**

Estimation of the branching ratio

For the Hawkes process the branching ratio is given by expression:

$$n = \int_0^\infty \varphi(t)dt$$

In particular, for the exponential kernel: $n = \alpha/\beta$

The Maximum Likelihood estimator:

$$\log L(\theta|t_1, \ldots, t_n) = -\int_0^T \lambda_t(t)dt + \int_0^T \log \lambda_t(t)dN(t)$$

In particular, for the exponential kernel:

$$\log L = -\mu T + n \sum_{t_i<T} \left(e^{-\beta(T-t_i)} - 1\right) + \sum_{t_i<T} \log \left(\mu + n\alpha \sum_{t_j<t_i} e^{-\beta(t_i-t_j)}\right)$$

Seasonality profiles

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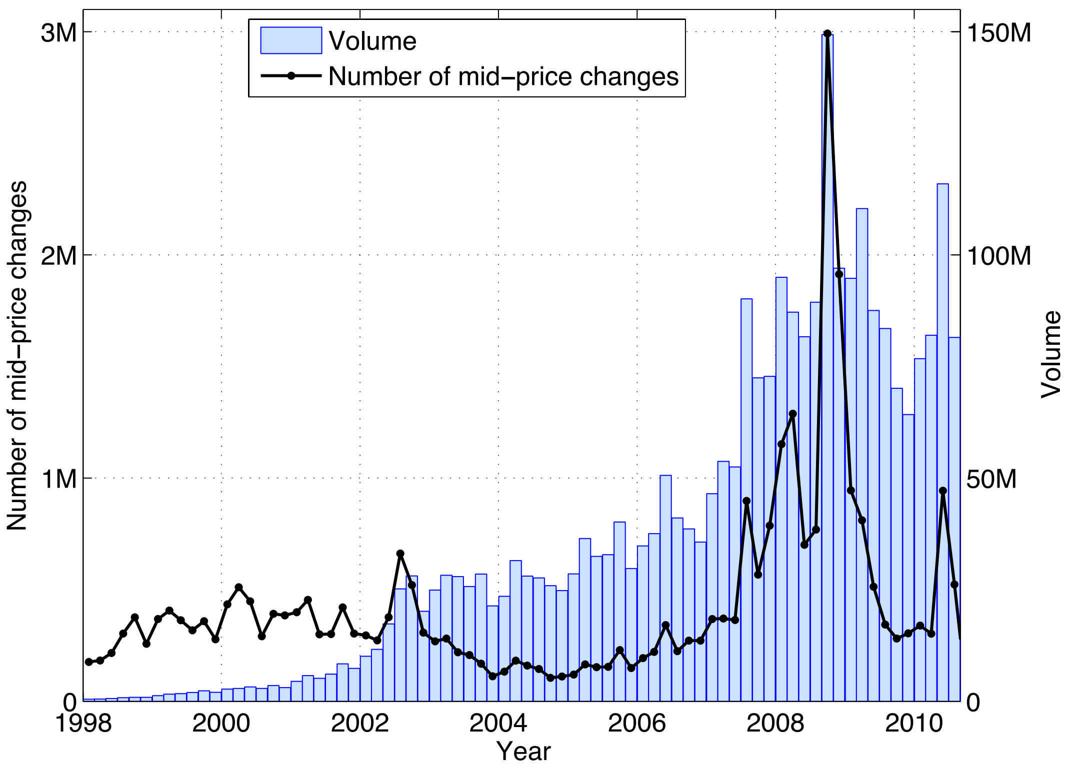
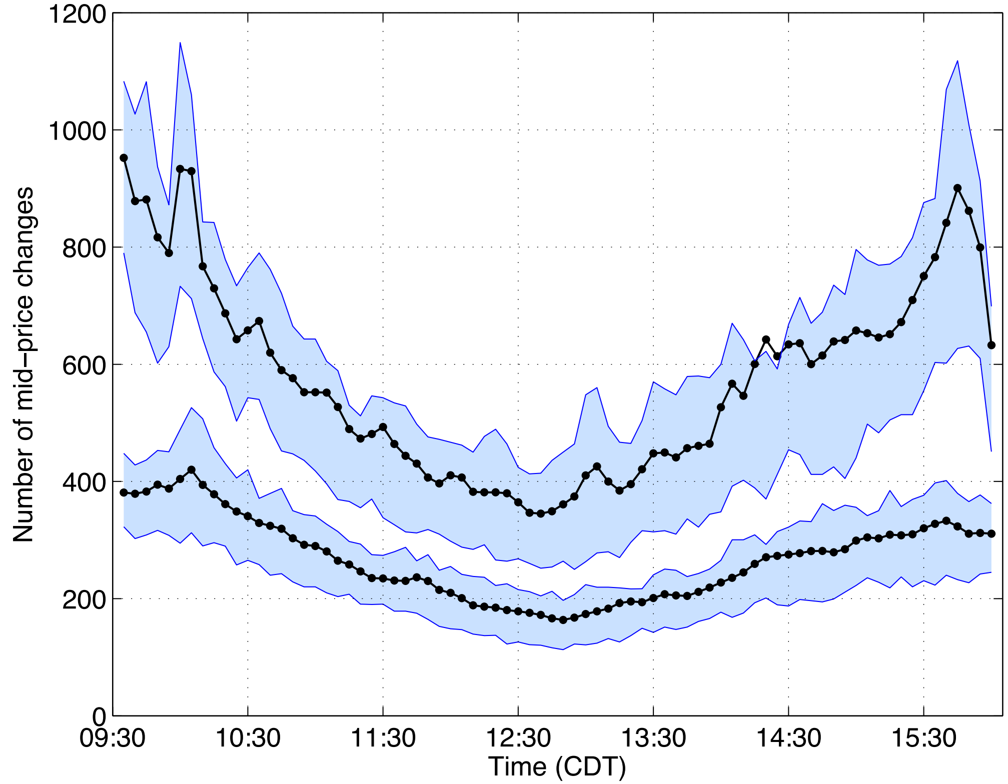


Average numbers of mid-price changes per day and average daily volume in 1998-2010



Average number of mid-price changes in 10 minutes interval during Regular Trading Hours in 2001 and 2009

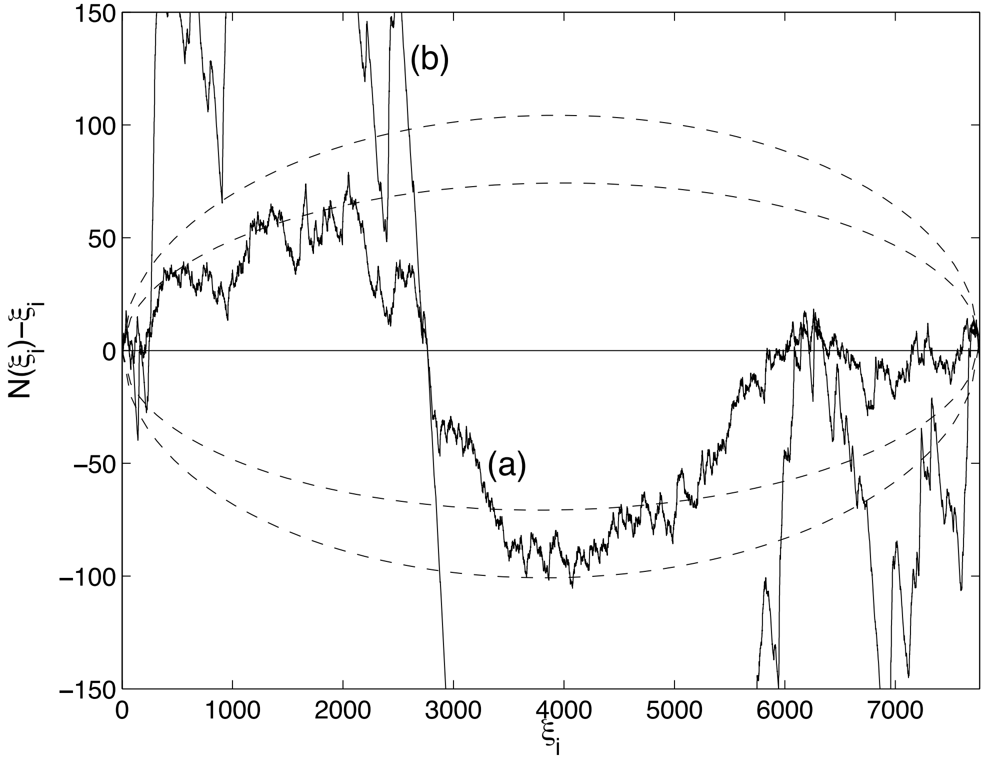
Residual analysis

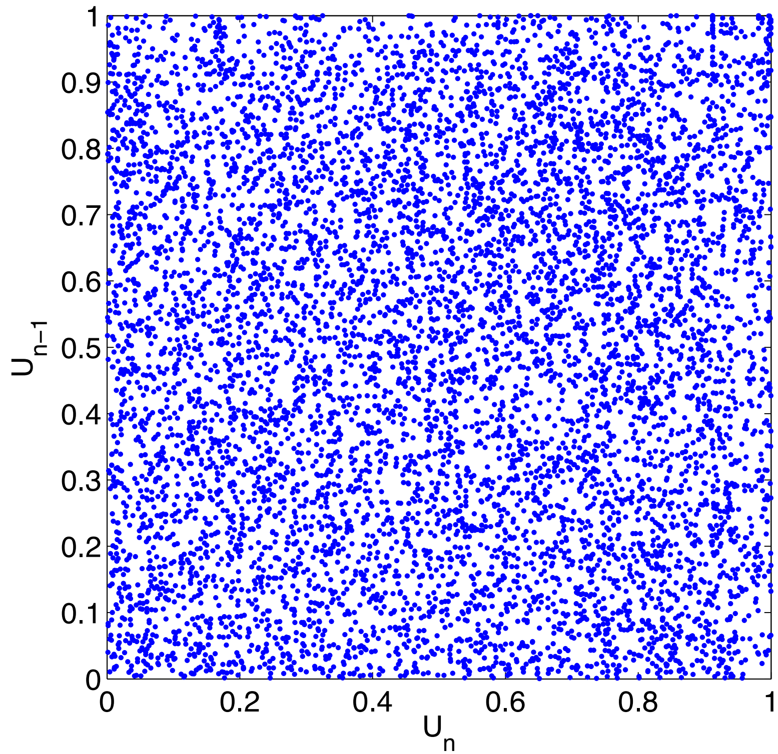
An important step of estimation procedure is **quality-of-fit** test.

For the Hawkes process it could be done with the residual analysis.

Residual process: $$\tau_k = \int_0^{t_k} \lambda_t(t)dt$$

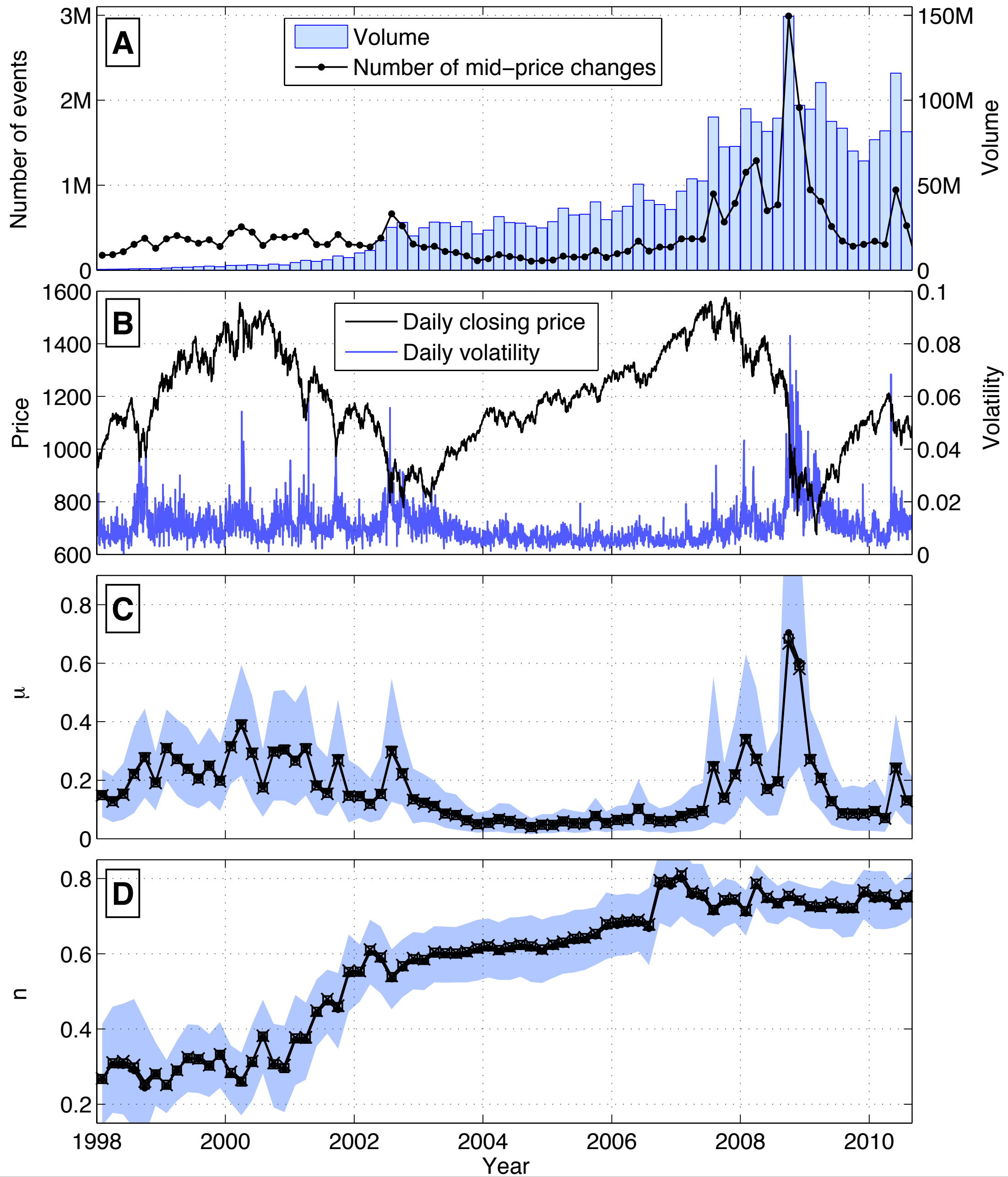
Eg. estimation within the period March 11, 2010 14:30-14:40:





$$U_k = 1 - \exp(\tau_{k-1} - \tau_k)$$

Estimation of reflexivity



- V. Filimonov, D. Sornette (2012) “Quantifying reflexivity in financial markets: towards a prediction of flash crashes”, submitted to PRE, http://arxiv.org/abs/1201.3572

Algorithmic and High-Frequency Trading

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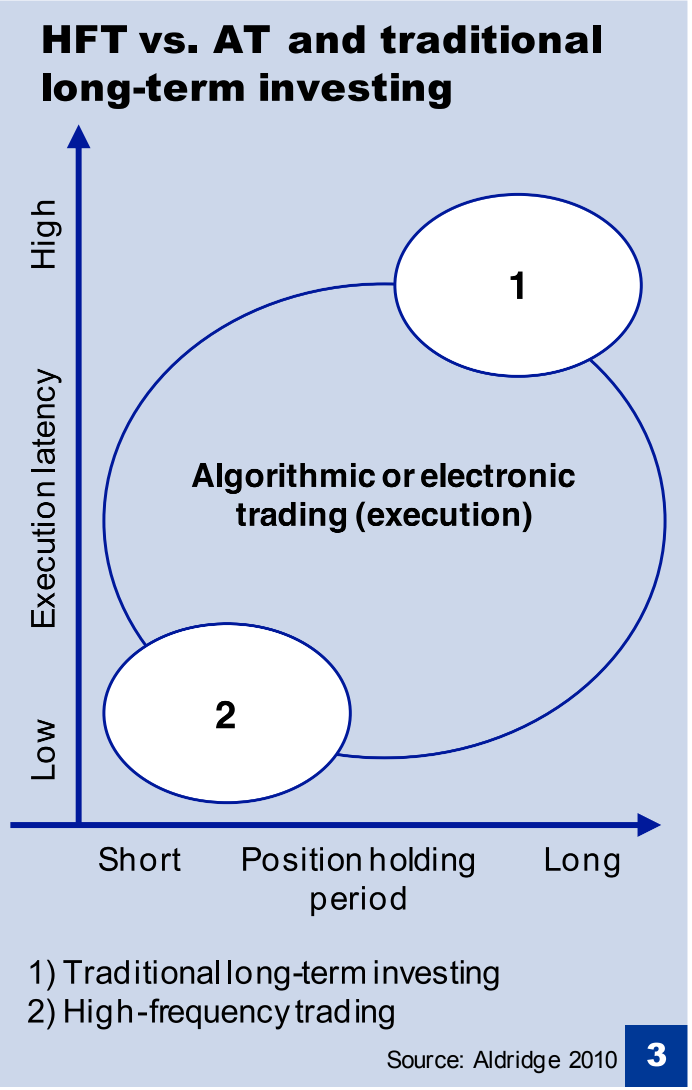

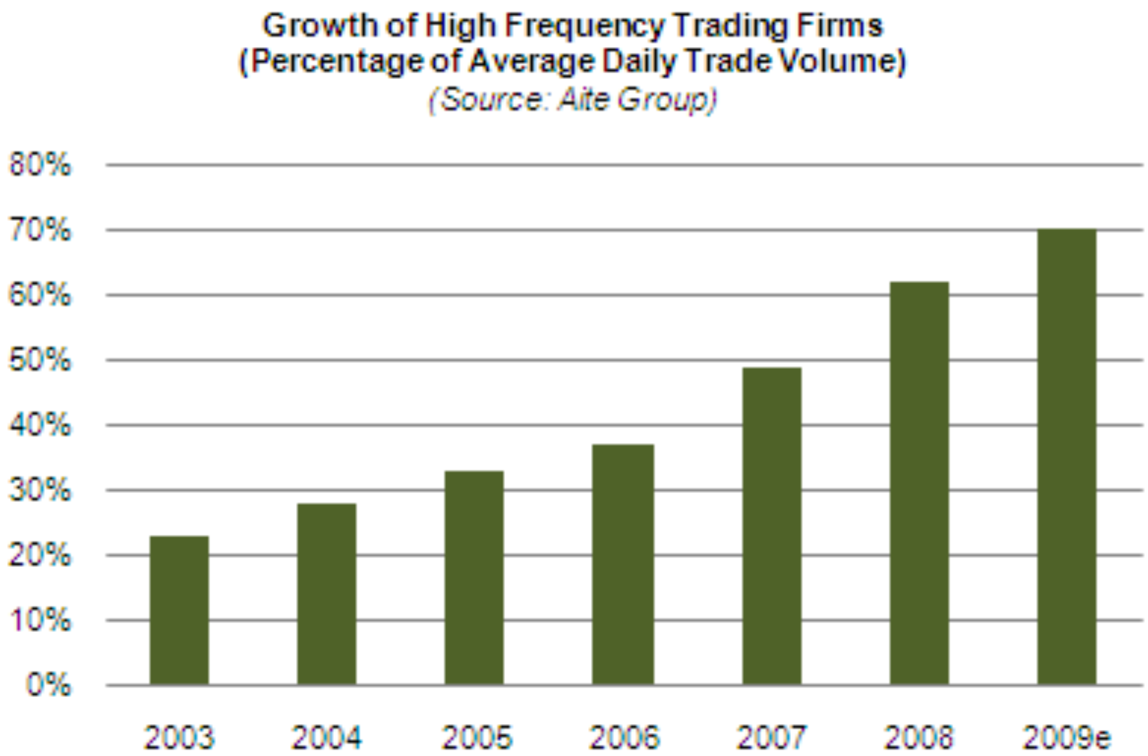

- "New World Order: the High Frequency Trading Community and Its Impact on Market Structure." The Aite Group Report (2009) http://www.aitegroup.com/Reports/ReportDetail.aspx?recordItemID=531
- I. Aldridge (2010) "High-Frequency Trading: A Practical Guide to Algorithmic Strategies and Trading Systems." John Wiley & Sons.

Flash-crash event

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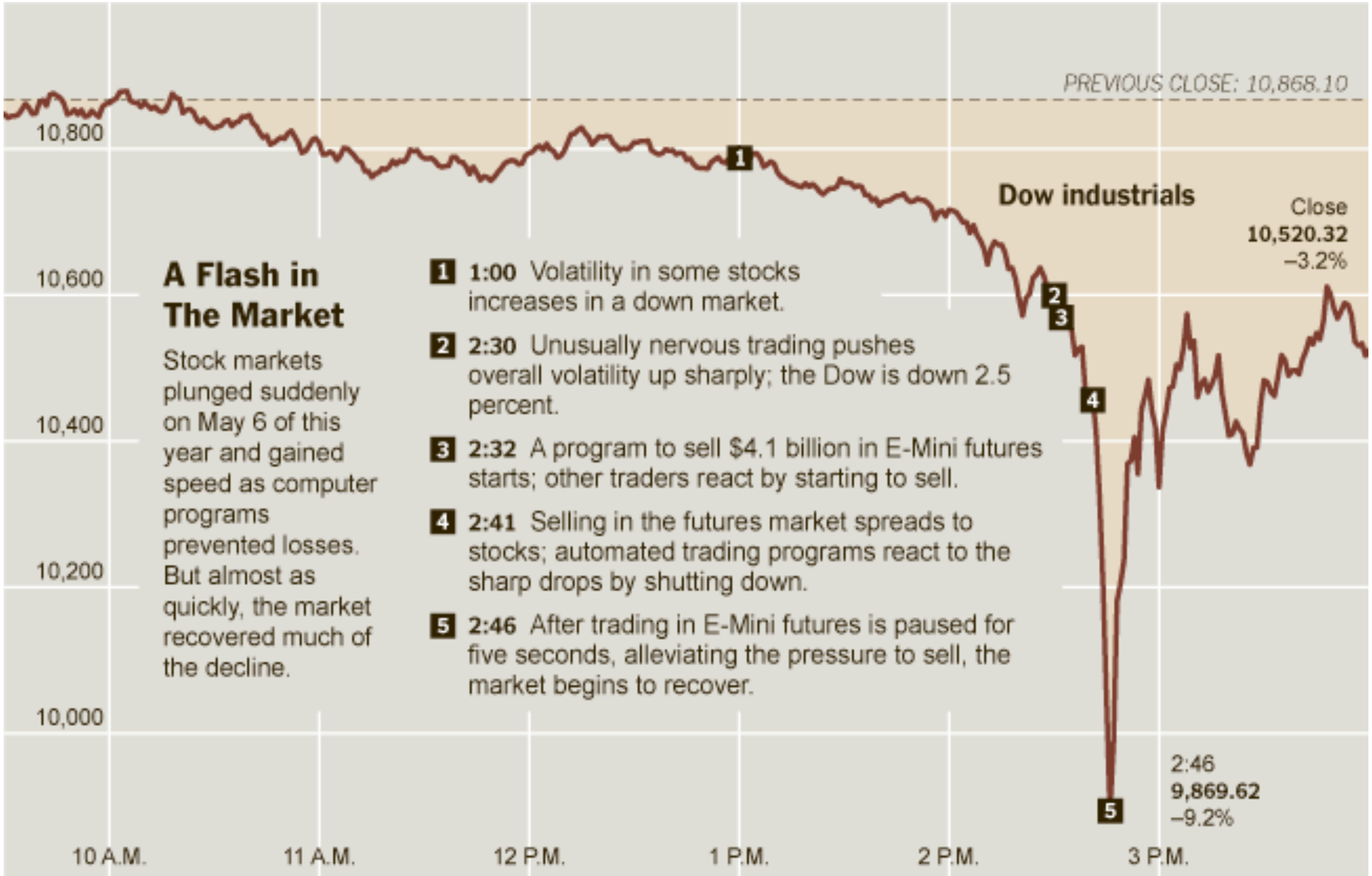

Sources: Bloomberg (Dow industrials); Securities and Exchange Commission THE NEW YORK TIMES

- “Findings Regarding the Market Events of May 6, 2010”: Report of the Staffs of the CFTC and SEC to the Joint Advisory Committee on Emerging Regulatory Issues (September 30, 2010).
- *Source of picture*: G. Bowley “Lone $4.1 Billion Sale Led to ‘Flash Crash’ in May”, New York Times (Oct. 2, 2010)

Exogenous vs endogenous shocks in HF

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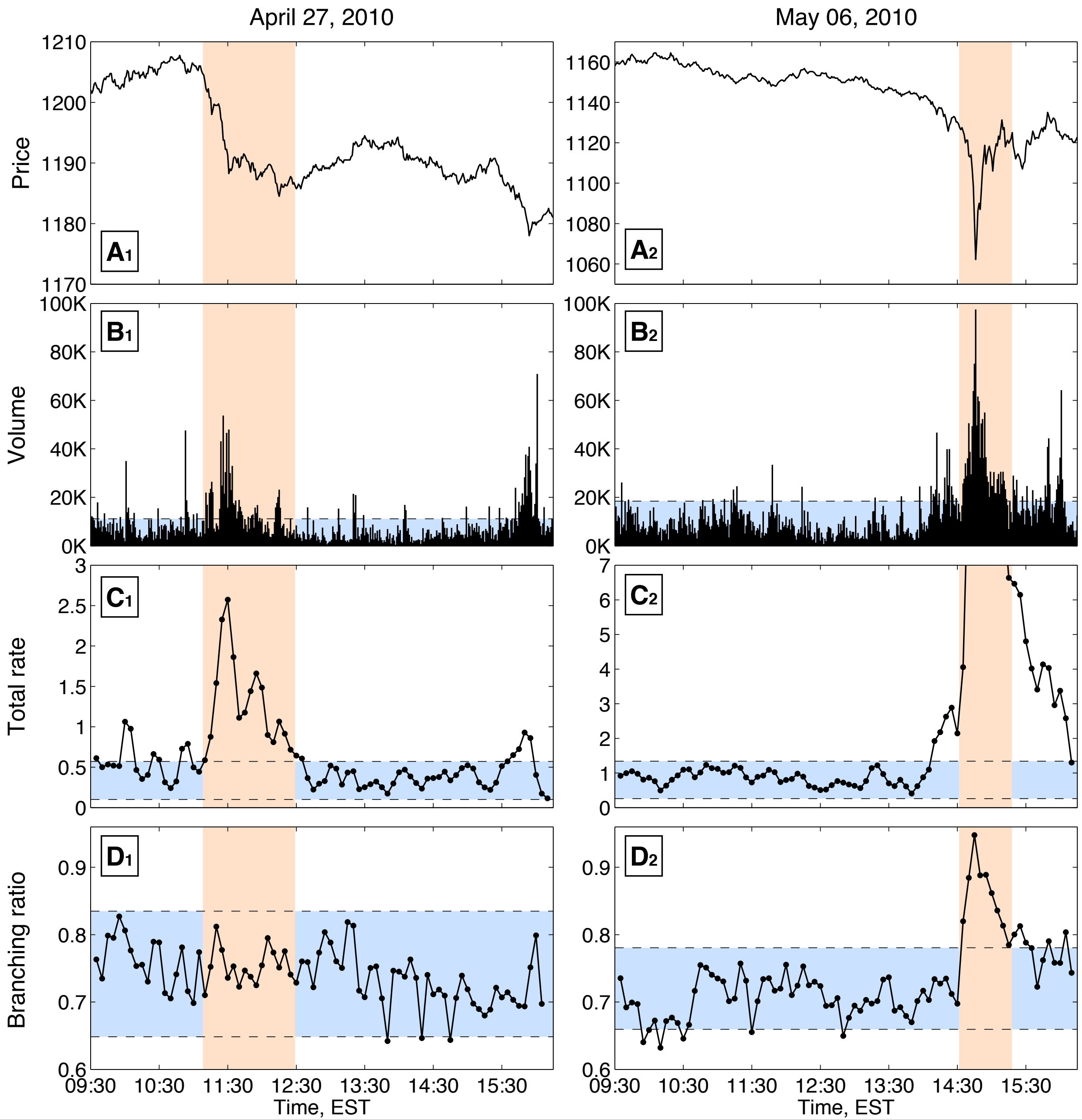

- V. Filimonov, D. Sornette (2012) "Quantifying reflexivity in financial markets: towards a prediction of flash crashes", submitted to PRE, http://arxiv.org/abs/1201.3572

Final remarks

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- In contrast to “neo-classical” theories, feedback mechanisms (reflexivity) play exceptionally important role in price dynamics.
- News plays a minor role in market volatility; most of price changes are result of internal feedback mechanisms. Due to the development of AT (and in particular HFT) endogeneity of price movements increased dramatically.
- The estimation of the branching ratio provides a novel powerful metric of endogeneity, which is much richer than standard direct measures of activity such as volume and trading rates.
- This measure allows real-time diagnostics of the market and distinguishing of exogenous (triggered by news) and endogenous (self-excited) events.