Evidence of microstructure variables nonlinear dynamics from noised high-frequency data

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Possibilities of chaotic dynamics

- Usually we suppose that microstructure variables have stochastic nature.

- On the other hand many nonlinear deterministic systems are known to produce trajectories which have statistical properties of stochastic series.

Possibilities of chaotic dynamics

- Simple example: autocorrelations for the tent map

$$x_t = \begin{cases} a^{-1} x_{t-1}, \ 0 \le x_{t-1} < a, \\ (1-a)^{-1}(1-x_{t-1}), \ a \le x_{t-1} \le 1. \end{cases}$$

  are the same as that of some first-order autoregressive process;

- when *a* is close to *0,5* autocorrelations are close to ‘white noise’ (Sakai & Tokumaru, 1980).

Possibilities of chaotic dynamics

- Can we consider that a financial series is a trajectory of some deterministic chaotic system?

- Advantages:
  - It is theoretically possible to forecast the exact future values for a short horizon;
  - During crisis the behavior of prices and other microstructure variables is chaotic rather than stochastic.

Chaotic or Stochastic?

How can we distinguish between ‘white chaos’ and stochastic white noise by looking at the trajectory?

– There’s no statistical test that has chaos as the null hypothesis

– Due to the presence of noise in microstructure data smoothing must be applied to the original signal (WaveShrink approach was used here).

But: problems with forecastability.

Data Set

- High-frequency dynamics of several liquid MICEX shares was examined (10 sec time step) for 6 month period (01.2006-06.2006).

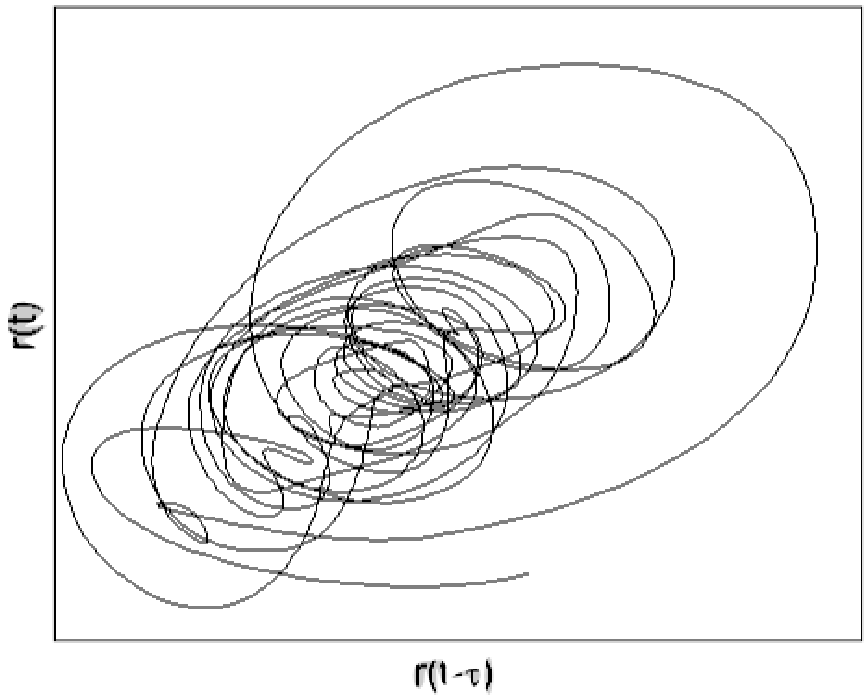
- Microstructure variables:

•**Price**	•**Return**
•**Price change**	•**Absolute price change**
•**Spread**	•**Relative spread**

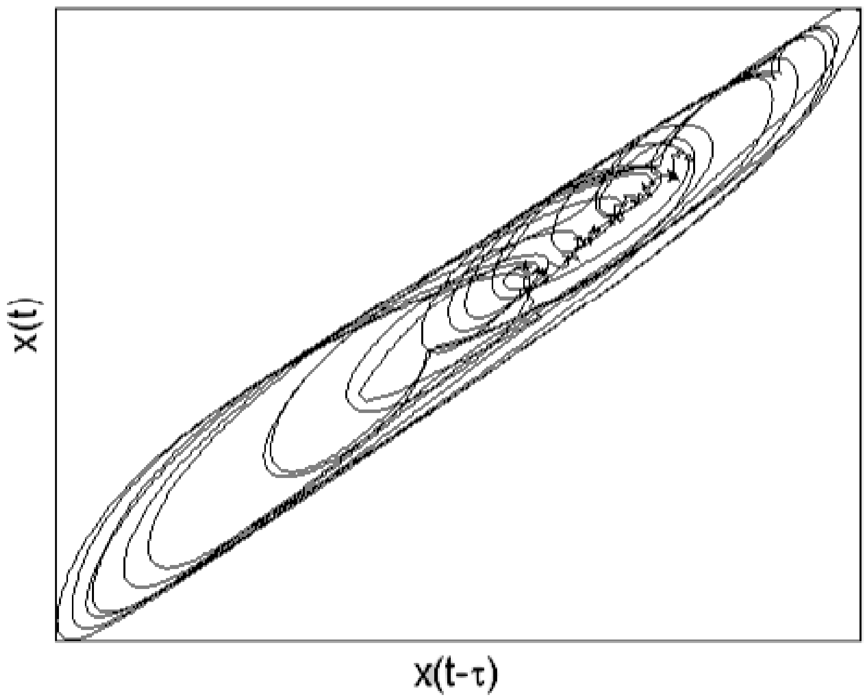
First evidence of deterministic structure

Let’s find out if there’s any non-random structure in smoothed microstructure series by looking at the dependence of lagged values:

Return r(t), $\tau = 20$



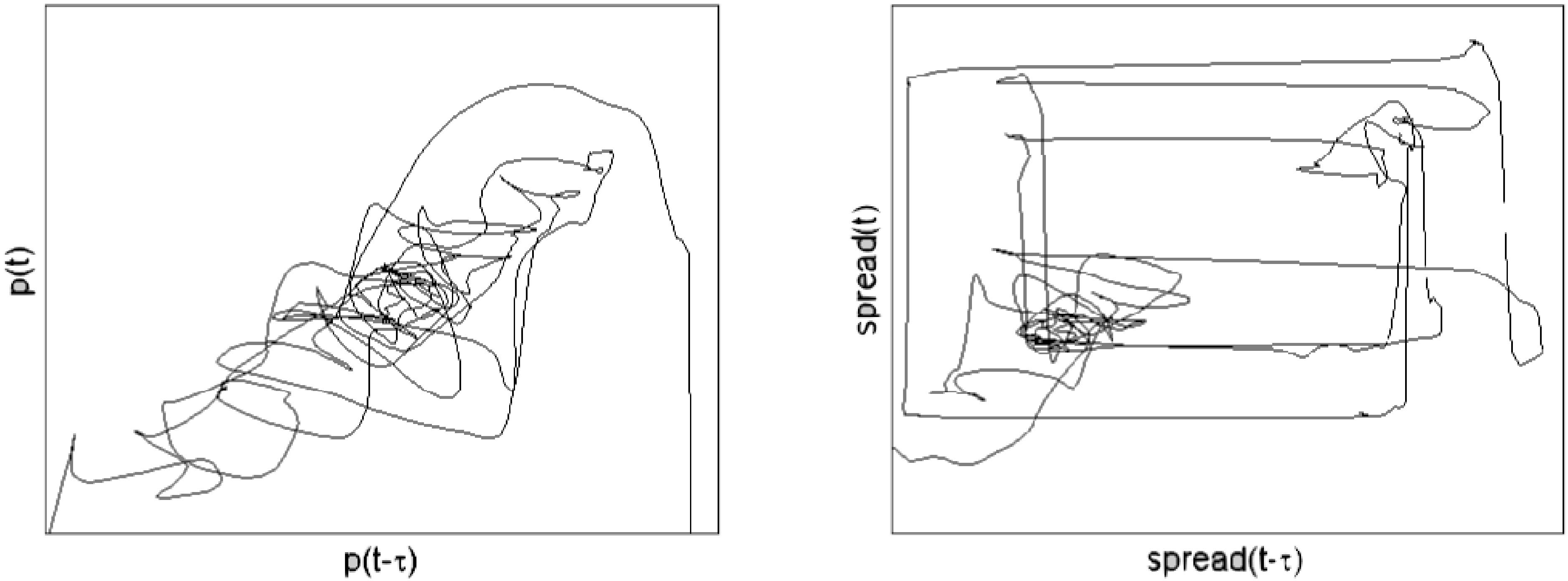
Mackey-Glass system, $\tau = 20$



First evidence of deterministic structure

Unlike returns, price change and relative spread we can't see any reasonable pattern for price and spread movement:

Price p(t), $\tau = 20$ Spread, $\tau = 20$



Correlation Dimension

- Grassberger & Procaccia (1983) presented a characteristic property of a variety of stochastic processes:

Given $x_1, x_2, \ldots x_N$, we construct $y_k = (x_k, x_{k-p}, \ldots x_{k-(M+1)p})^{M\times 1}$

Then correlation integral $C_M(\varepsilon) = \dfrac{number\ \ of\ \ pairs\ \ (y_i, y_j):\ \|y_i - y_j\| < \varepsilon}{total\ \ number\ \ of\ \ pairs\ \ (y_i, y_j)}$

satisfies the following law for small ε :

$$C_M(\varepsilon) \approx \varepsilon^{D_M}$$

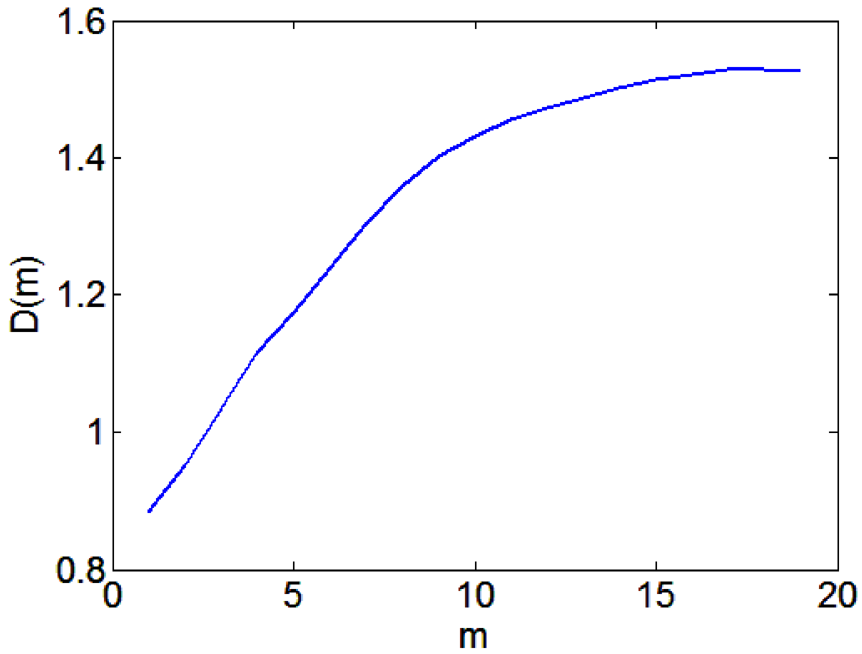
Correlation Dimension

Criterion: for many stochastic trajectories $D_M \xrightarrow[M\to\infty]{} \infty$

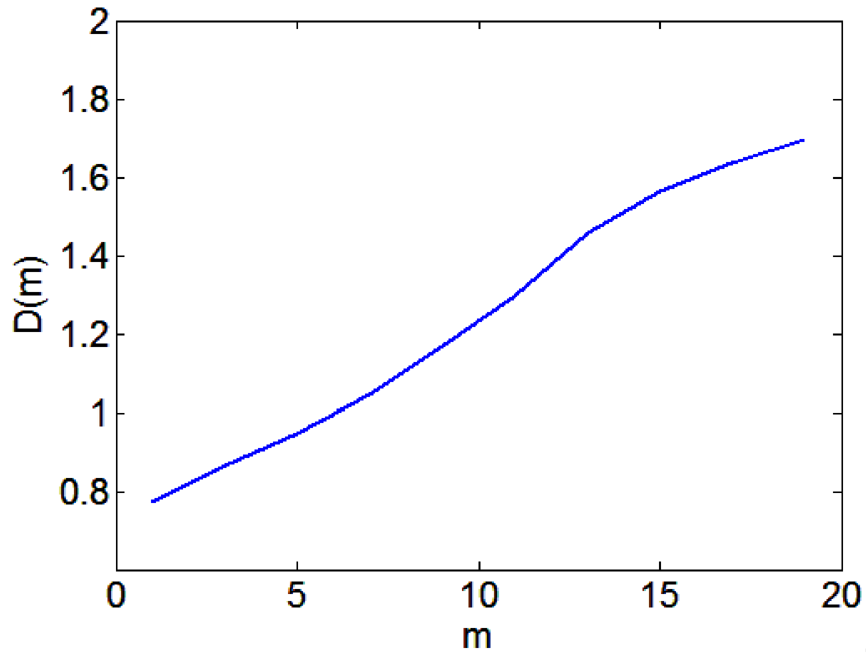
If saturation is observed $(D_M \xrightarrow[M\to\infty]{} D')$ then trajectory is very likely generated by a deterministic system.

$(2D'+1)$ - its dimension.

D_M for relative spread

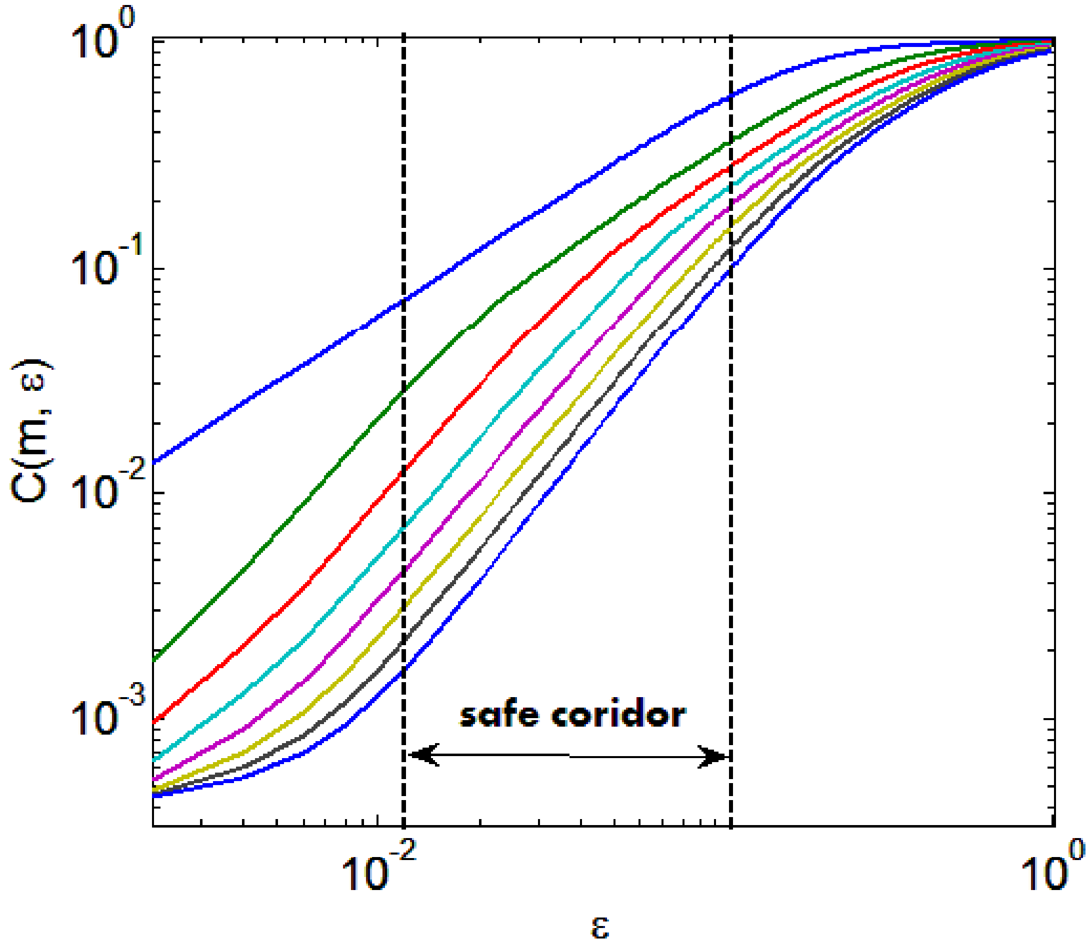


D_M for spread



Correlation Dimension

The main problem is choosing the appropriate ε range to estimate D_M :



BDS Test for Nonlinearity

- Brock, Dechert & Scheinkman (1986) presented a statistical test for nonlinearity in data. Linear model is fitted to series, then BDS test is applied to residuals using a null of i.i.d.

- Noticed problems (Liu, Granger & Heller, 1992):
  - Rejection of null does not imply chaos but some (maybe stochastic) nonlinearity,
  - BDS power varies depending on linear model, it has less power for nonlinear MA models.

Evidence of Forecastability

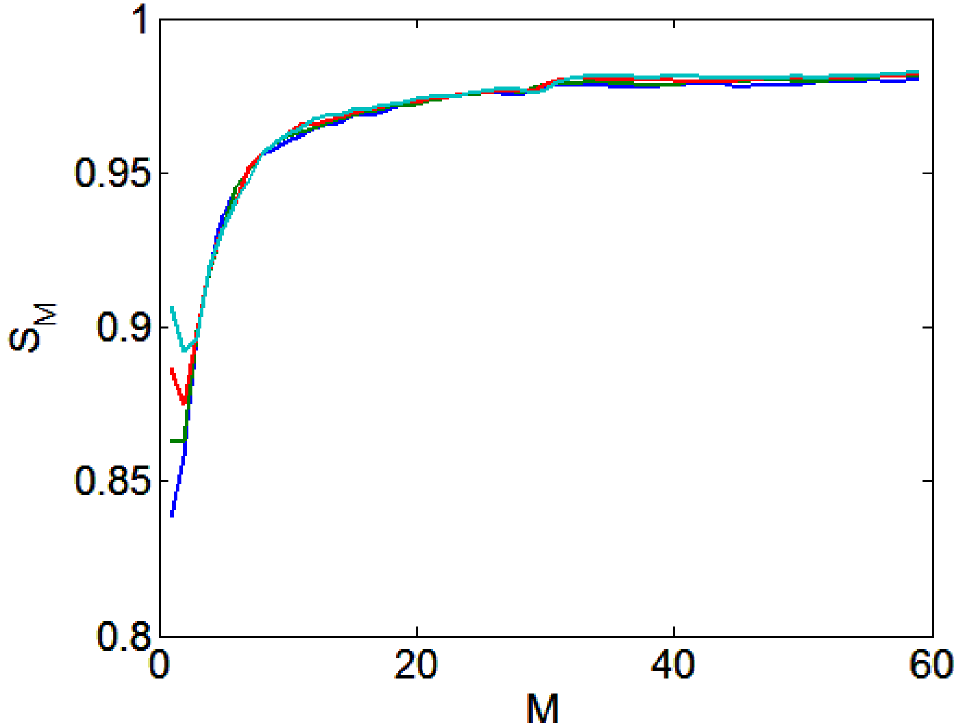
- Scheinkman & LeBaron (1989) prove that:

  $S_M(\varepsilon) = \dfrac{C_{M+1}(\varepsilon)}{C_M(\varepsilon)}$ is the conditional probability that two states of the system are close given that their past *M* histories are close

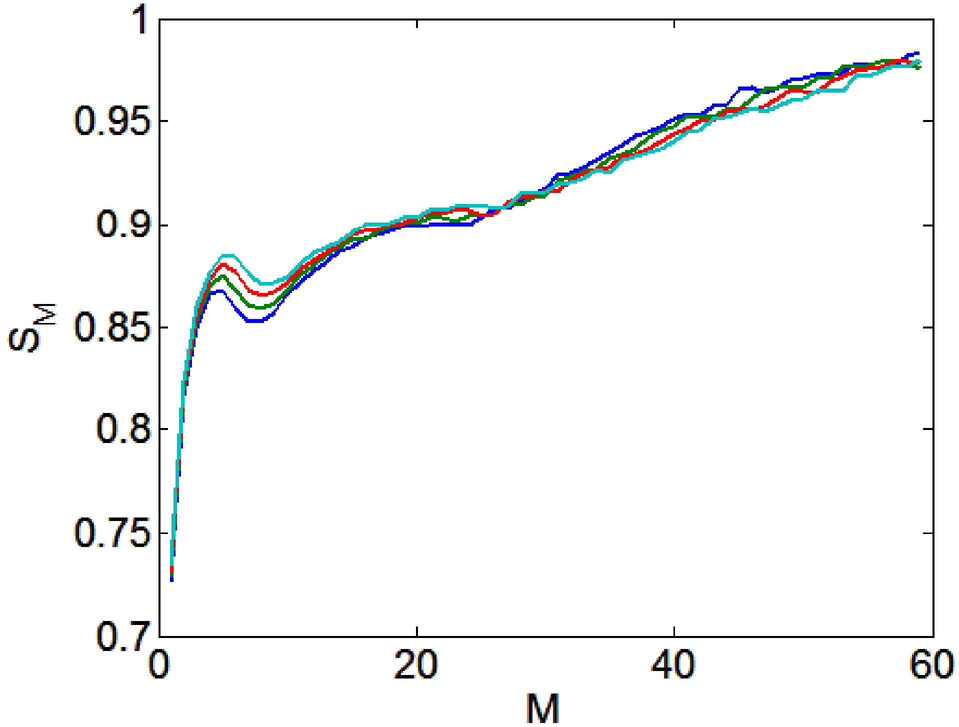
- Implementation:
  - If states are independent then $S_M(\varepsilon)$ does not depend on *M*;
  - If past values of the series help predict future values, $S_M(\varepsilon)$ will tend to increase with *M*.

Evidence of Forecastability

$S_M(\varepsilon)$ **for spread**
(functions for 4 different ε **)**



$S_M(\varepsilon)$ **for returns**
(functions for 4 different ε **)**



Evidence of Predictability

	$S_M(\varepsilon)$
Returns	increases
Relative Spread	increases
Spread	converges to 0,9
Price	converges to 0,96
Price Change	increases
Absolute Price Change	increases

Recurrence Plots

- For the past decade new developments for recurrence plots were achieved.

- Recurrence plot indicates if different states of the dynamic system are close or not:

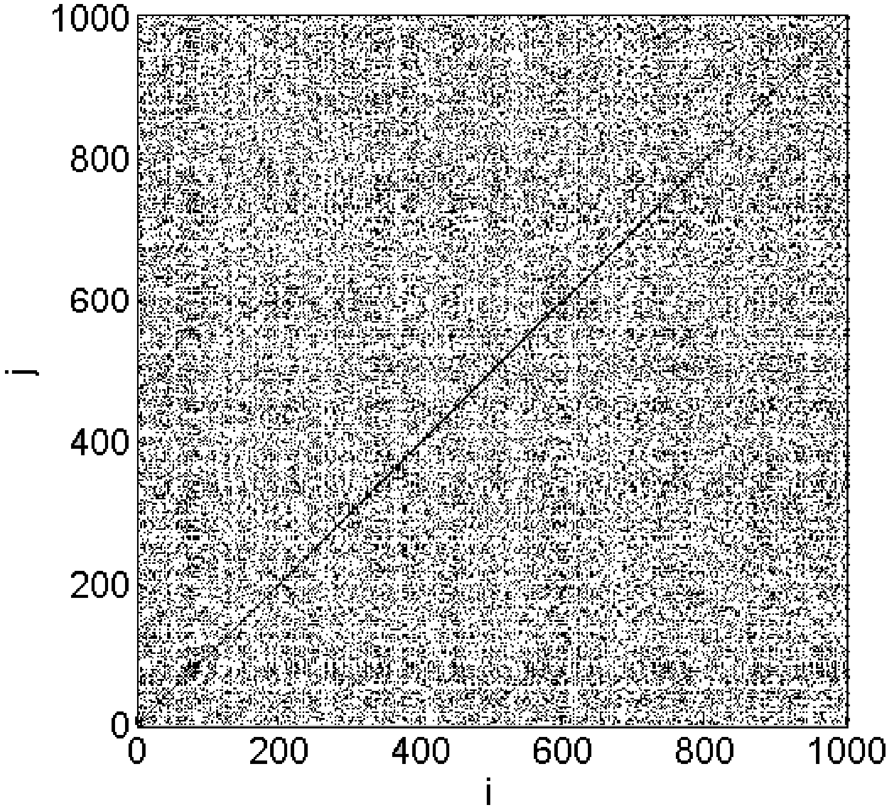
$$R_{\varepsilon}(i, j) = \begin{cases} 1, & if \ \left\| x_i - x_j \right\| < \varepsilon, \\ 0, & otherwise. \end{cases}$$

Recurrence Plots: Advantages

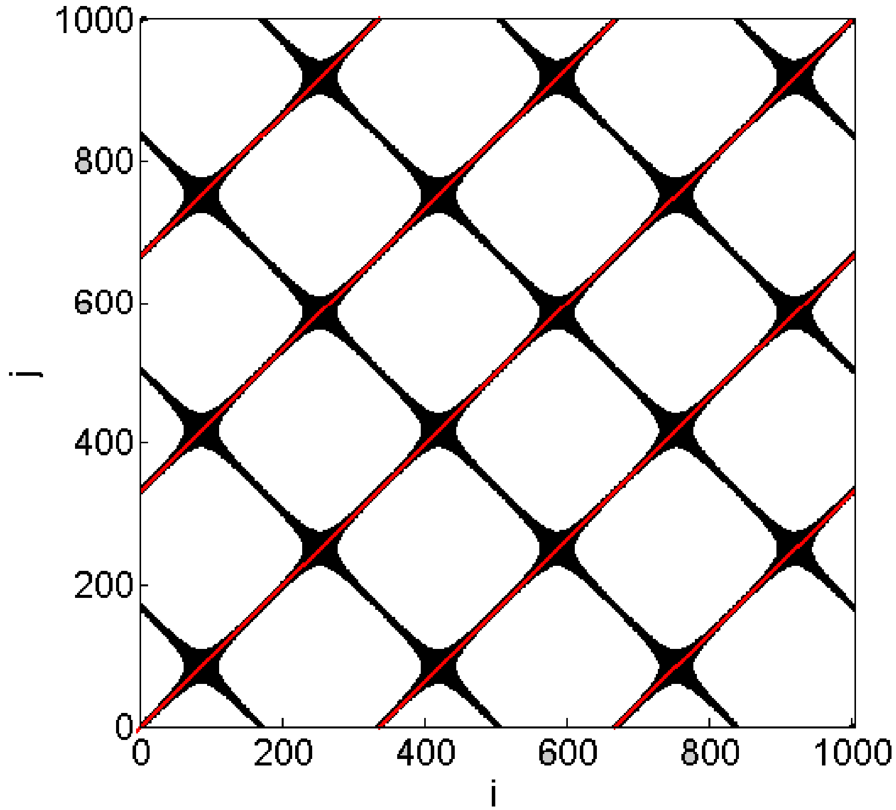
- RP allows simple and reasonable visualization of n-dimensional dynamics.
- Thiel et al (2004) developed RP-based approach to estimating correlation dimension and other dynamic invariants without choosing embedding parameters *(p,M)*
- Presence of nonlinear structure in data can be noticed even without quantification.

Recurrence Plot: white noise vs predictable system

Recurrence plot for i.i.d. $x(t)$

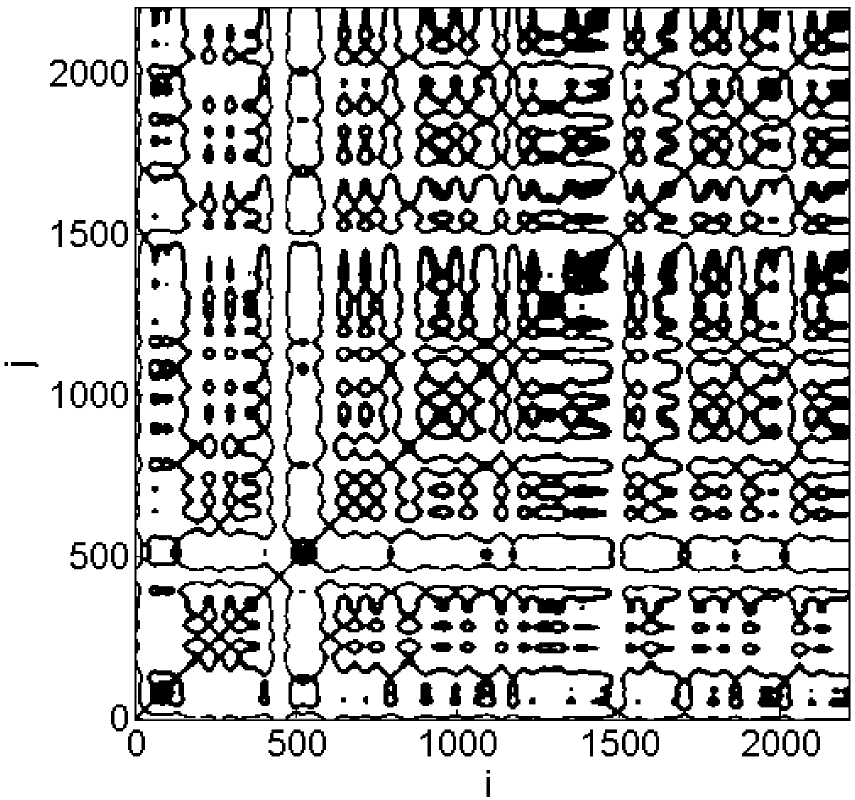


Recurrence plot for $x(t) = \sin(6\pi t)$

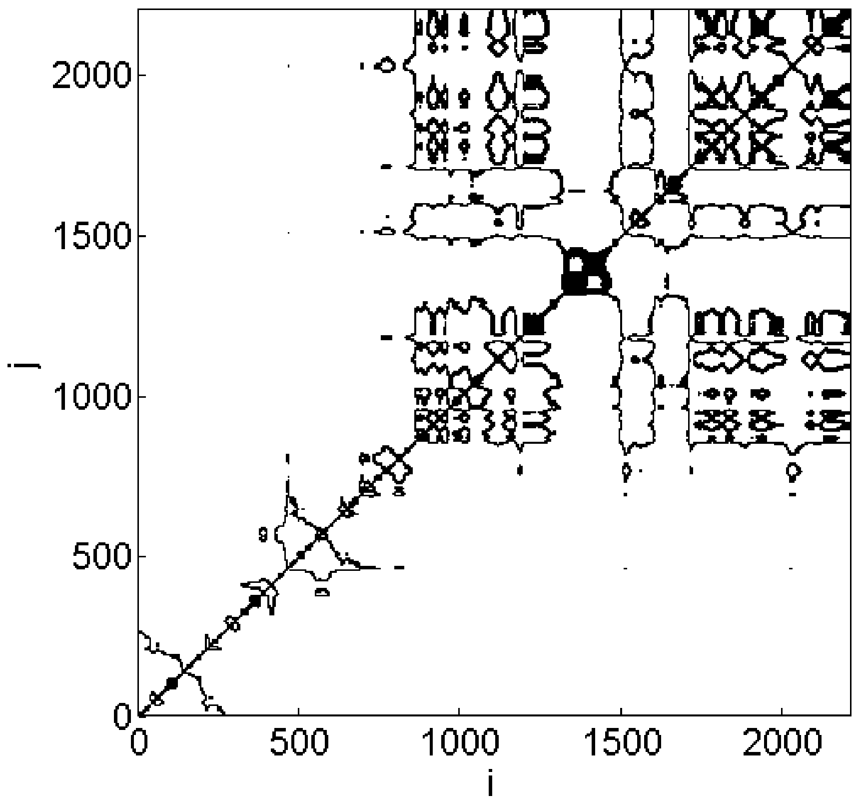


Recurrence Plot: real systems

Recurrence plot for returns



Recurrence plot for price



Recurrence Quantification Analysis

- Total amount of diagonal lines in RP reflects the predictability and determinism of the trajectory.

- Real data have quite complicated RPs, so several numeric measures of determinism were proposed (quantification analysis).

Let *P(L)* be the probability to find a diagonal line of length *L* in the RP.

Recurrence Quantification Analysis

Name	Definition	Interpretation
Recurrence rate (*RR*)	Percentage of black points in RP	Correlation integral
Determinism (*DET*)	Percentage of black points which are part of diagonal lines of at least length *L*.	Measures predictability
Entropy (ENTR)	Shannon entropy of the distribution of diagonal lines *P(L)*	Quantifies the complexity of the deterministic structure
Laminarity (LAM)	Same as DET but for vertical lines	Quantifies the occurrence of laminar states
Trapping Time (TT)	Mean length of vertical lines	Measures the mean time that the system sticks to a certain state

Results of RQA for microstructure series

	RR	DET	ENTR	LAM	TT
Return	6,23%	0,91%	0,19	56%	16
Relative Spread	6,43%	0,51%	0,23	73,92%	18
Spread	8,19%	0,03%	0,68	78,38%	21
Price	6,95%	3,7%	0,28	78,02%	21
Price Change	5,91%	0,17%	0,32	58,92%	16
Absolute Price Change	7,91%	0,39%	0,26	69,81%	17

What are the results?

- **Returns, price changes, relative spread** show signs of complex nonlinear underlying structure, thus random walk model isn't appropriate for them.

  Scheinkman-LeBaron procedure shows that history of these variables helps to predict future values.

- **Price** and **spread** dynamics show purely stochastic behavior or the amount of noise is too great. Future values do not depend on history.

- RQA shows the presence of determinism in **returns, relative spread** and **absolute price change** dynamics, but no determinism for **spread**.

References

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Questions?